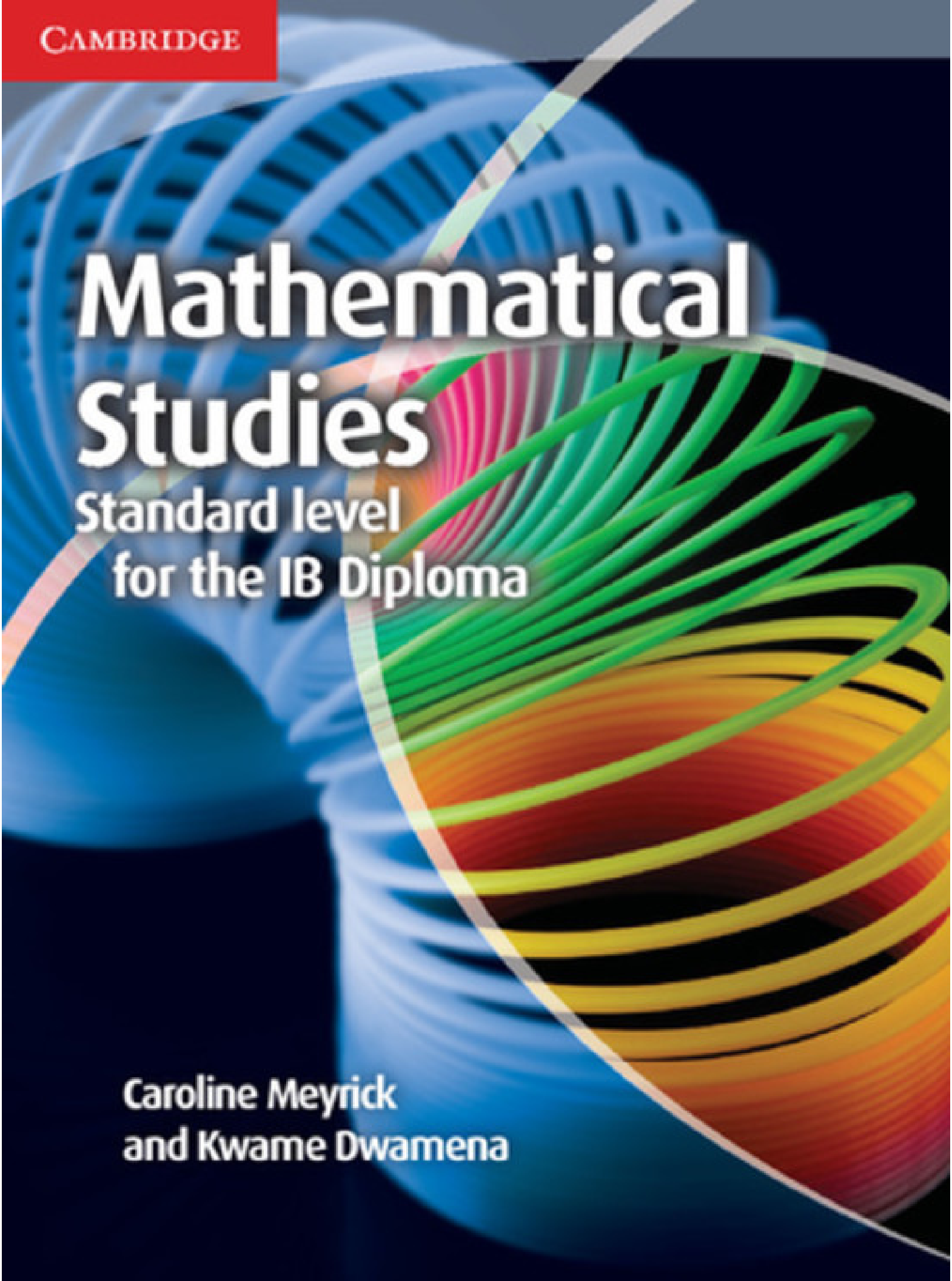
CAMBRIDGE
Mathematical Studies
Standard level for the IB Diploma
Caroline Meyrick and Kwame Dwamena

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

www.cambridge.org
Information on this title: www.cambridge.org/9781107691407

First published 2013

Printed in the United Kingdom by Latimer Trend

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-69140-7 Paperback

Cover image: © sharon zilberczvig / Alamy

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Introduction

This book has been written for students and teachers taking the Mathematical Studies SL course of the International Baccalaureate®. The syllabus is divided into seven topics and the book follows this structure, with each topic divided into chapters within that theme. The syllabus is designed to emphasise the practical mathematics that will enable students to continue to use their learning at university, in training courses, or when working, and the book encourages this. Explanations, worked examples and exercises are all firmly rooted in ideas that might be met in the course of normal experience.

As this book will be used by students from widely differing mathematical backgrounds, we have included fast forward and rewind indicators that mean it is not necessary to follow the order of the book exactly as it has been written; teachers will be able to integrate appropriate sections of the book with their own schemes of work:

The rewind indicators will refer the reader back to another section in the book if any prerequisite information is required to complete a section.

The fast forward indicators inform the reader where topics will be covered in more detail later in the book.

The concepts in Topic 6, Mathematical models, have been extended throughout the text. Problem solving is a vital skill, and confidence with this is achieved by students who realise that they can approach solutions from different perspectives. Worked examples aim to show that there might be several ways to find an answer: through drawing diagrams, working from first principles, through algebra or the use of technology.

It is important to understand that the Graphical Display Calculator (GDC) is fundamental to the course, and is allowed not only in both examinations, but also when students are working on their project. Where appropriate, all examples requiring the use of the GDC are illustrated by screen shots, and if students need additional support, the GDC chapter at the end of the book gives more detailed instruction of how to use their GDC. The calculators used in the writing of this book are the Casio *fx-9750G11* and the Texas TI-84 Plus Silver Edition. While many students might find they need to use slightly different key strokes from those that are given, the examples are close to those in the authors' experience of the many different versions of GDC that students have shown them.

How to use this book

Book structure

Each topic begins with an introduction that emphasises the place of that topic within mathematical history, practical experience, or with some information about internationally recognised mathematical people. It also lists any prior knowledge that is required, and any previous chapter that needs to be completed before this topic is started.

Each chapter of the book follows a pattern designed to be helpful to students from different national educational programmes, and familiar with different mathematical traditions. Like the topics, each chapter starts with an introduction that gives some context to the chapter content in terms of mathematical history, practical experience, or a historical person. There are also 'In this chapter you will learn' panels that list the learning objectives of the chapter.

The chapters contain numerous worked examples throughout. These are arranged with the formal written mathematics on the right-hand side (in blue font), and the thought processes or hints on the left-hand side in speech bubbles. Initially the worked examples can be followed using the ideas in the speech bubbles. For revision, the hints can be covered up, and the student can concentrate on the formal written mathematics that they will need in their examinations.

First, draw a diagram and label it with the information from the question.

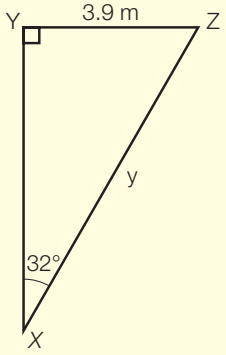
Then identify the ratio needed and solve the equation

To find XZ, you are using the opposite side and the hypotenuse. Choose SOH.

Using Pythagoras' Theorem

Worked example

Formal working, e.g.:



a) $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin 32° = \frac{3.9}{y}$

$y = \frac{3.9}{\sin 32°}$

$y = 7.36\text{m}$

b) $XY^2 = XZ^2 - YZ^2$

$= 7.36^2 - 3.9^2$

$= 6.24\text{ m}$

At the end of each sub-section of a chapter is an exercise. This allows the student to check their knowledge and understanding of the chapter and the methods learned so far. These are in the format of drill-like questions that act as a quick self-assessment of progress.

Each chapter finishes with summary list of what the student should know once they have completed the chapter. This can be tested using the Mixed Examination practice questions that follow. These include a selection of exam-style questions written by the authors in the style of IB examination questions, and some real exam questions taken from past IB Mathematical Studies papers.

The questions are designed to make sure that students understand the mathematics studied in that chapter and how to use it. For some questions, particularly in the latter half of the book, students might be expected to use

concepts from more than one topic, which is a common theme for IB questions. It is very important that students have practised combining ideas and techniques before they attempt any past examination papers.

Supporting panels and icons

Within each chapter there are various support panels and icons to guide the student, or encourage them to look beyond their immediate studies.

Learning links

These are designed to link the course with more formal mathematical processes that might have been learned in the past. They **do not** have to be used because they are not on the syllabus. However, if the student wishes to see the connections between this syllabus and previous learning, the link is easy to follow.

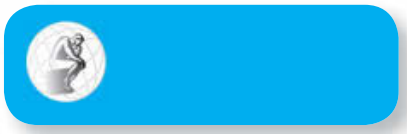
Help with using the text

These are integrated throughout the book, often recalling the student's attention to a particular point in the text or a worked example, or reminding them of a general point of good mathematical practice.

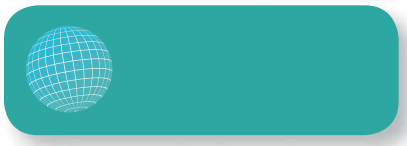
These points allow students to easily go backwards or forwards to any section if they need reminders.

This book is based on a specific syllabus, but should not be seen solely as a guide to the final examinations. However, for all students it is reassuring to have common mistakes and misunderstandings pointed out, so that their examinations can be approached with confidence.

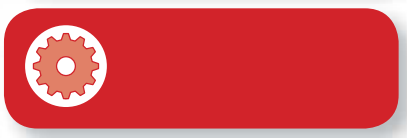
Icons linking in the wider IB Diploma ethos



The Theory of Knowledge is central to the IB Diploma. These panels aim to link questions met in the TOK course with ideas explored in the Mathematical Studies SL course.



These panels link to other subject areas providing cross-curricular support as well as links to wider applications of mathematics in the practical world.



Critical thinking is part of mathematics as it is a tool that can be used in many contexts and cultures. These panels highlight issues about the application of mathematics in the real world and to other subject areas. It also includes links to mathematical history, which places today's mathematics in context and shows the breadth and spread of mathematical ideas.

The internal assessment

All Mathematical Studies students complete a project as part of their diploma course. It is worth 20% of their final mark, and needs to be given approximately 20% of the time allowed for an SL subject.

Throughout the book, some comments are made about ideas and techniques that may be useful in projects. There is also a chapter at the end of the book giving advice in more depth, concentrating in particular on the different assessment criteria, giving advice about how to make sure that these are understood and correctly interpreted.

Use of technology

The use of technology is encouraged on this course. This does not need to be restricted to the use of the GDC. Personal computers (PCs, Mac, Laptop, tablets etc.) can also reinforce and encourage understanding; this can be through using commercial geometric or statistical packages, as well as programmes that students already know, such as spreadsheets. There are also many websites and videos online that can be very helpful; though students should be cautious about trusting the site before using them.

Online material

There are two practice examination papers available online at

education.cambridge.org/mathsstudiespaper1

education.cambridge.org/mathsstudiespaper2

Paper 1 has fifteen questions, and each question is marked out of six. The paper takes 90 minutes, so students should aim to gain one mark a minute.

Paper 2 has six questions, and the mark for each question can vary depending on its length and complexity. The total time for the paper is 90 minutes, and the total marks are also 90, as in Paper 1. In paper 2, if a question is marked out of 15, the student should aim to complete it in fifteen minutes.

We hope you find the Mathematical Studies SL course both interesting and accessible. Mathematics is a unique combination of understanding and practice, and we have written the text and exercises to help you with both sides of the subject. If you can build on the knowledge you already have, you should be able to move forward with a useful background in mathematics that will be relevant in future studies and your workplace.

Kwame Dwamena

Caroline Meyrick

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Text

p2 Ada Lovelace quotation: 'Note G' from Ada Lovelace's translation and commentary of a 1842 document called Sketch of the Analytical Engine by L.F. Menabrea (information sourced from www.allonrobots .com/ada-lovelace.html); p194 Data: the consumption of softwood throughout the UK for 2000-2009 www.forestry.gov.uk/forestry, p195 Data: average pump price of fuel per litre in 194 countries in 2010 www.data.worldbank.org; p277 U.S. field production of crude oil between 1920 and 2010 U.S. Energy Information Administration (2012); p592 Extract from Simon Singh's book *Fermat's Last Theorem* Simon Singh, *Fermat's Last Theorem* (London: Fourth Estate, 1997); p592 Quote of a saying of mathematician Hugo Rossi Hugo Rossi, *Notices of the American Mathematical Society*, (Vol 43, Number 10, 1996); p613 Quote from Ian Stewart's book *Seventeen Equations that Changed the World* Ian Stewart, *Seventeen Equations that Changed the World (*Profile Books,2012); p232 1899 'Everything that can be invented has been invented' A misquote attributed to Charles Holland Duell, Commissioner of the US Patent Office, 1899; p232 1943 'only be a world market for maybe five computers' Commonly attributed to Thomas J. Watson, Chairman of International Business Machines (IBM), 1943; p232 Quote from Richard Brealey and Stewart Myers, *Principles of Corporate Finance* 1988 Richard Brealey and Stewart Myers, *Principles of Corporate Finance* (McGraw Hill,1988).

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