

CAMBRIDGE

Mathematical Studies

Standard level
for the IB Diploma

Caroline Meyrick
and Kwame Dwamena

Mathematical Studies

**Standard level
for the IB Diploma**

**Caroline Meyrick
and Kwame Dwamena**

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

www.cambridge.org
Information on this title: www.cambridge.org/9781107691407

© Cambridge University Press 2013

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2013

Printed in the United Kingdom by Latimer Trend

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-69140-7 Paperback

Cover image: © sharon zilberczvig / Alamy

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate. Information regarding prices, travel timetables and other factual information given in this work is correct at the time of first printing but Cambridge University Press does not guarantee the accuracy of such information thereafter.

NOTICE TO TEACHERS

Worksheets and copies of them remain in the copyright of Cambridge University Press and such copies may not be distributed or used in any way outside the purchasing institution.

Contents

Introduction	vii
Acknowledgements	xi

Topic 1: Number and algebra 1

1 Number 2	3 Arithmetic and geometric sequences and series 65
1.1 Different types of numbers 2	3.1 Arithmetic sequences 65
1.2 Approximation and estimation 8	3.2 Arithmetic series: the sum of an arithmetic sequence 79
1.3 Expressing very large and very small numbers in standard form 22	3.3 Geometric sequences 90
1.4 SI units 27	3.4 Geometric series: the sum of a geometric sequence 98
2 Solving equations 38	4 Financial mathematics 110
2.1 Linear equations 38	4.1 Currency conversions 110
2.2 Pairs of linear equations 46	4.2 Compound interest 116
2.3 Quadratic equations 53	4.3 The GDC and financial mathematics 122
	4.4 Inflation 129
	4.5 Depreciation 132

Topic 2: Descriptive statistics 140

5 Classification and display of data 141	6 Measures of central tendency 182
5.1 Classifying data 141	6.1 Finding the median for simple data 184
5.2 Simple discrete data 145	6.2 Finding the mean for discrete and continuous data 185
5.3 Grouped discrete data 147	6.3 Identifying the mode or modal class 196
5.4 Grouped continuous data 149	6.4 Comparing the median, mean and mode 196
5.5 Mid-interval values and upper and lower boundaries 152	7 Measures of dispersion 206
5.6 Frequency histograms 155	7.1 Range and interquartile range 207
5.7 Cumulative frequency 160	7.2 Standard deviation 214
5.8 Box and whisker diagrams 169	7.3 Using the interquartile range and standard deviation to make comparisons 220

Topic 3: Logic, sets and probability

232

8	Set theory and Venn diagrams	233	10	Probability	288
8.1	Basic concepts of set theory	234	10.1	Introduction to probability	288
8.2	Venn diagrams with numbers	236	10.2	Sample space diagrams	290
8.3	Applications of set theory and Venn diagrams	239	10.3	Calculating probability and the expected value	294
8.4	Venn diagrams with three sets	242	10.4	Mutually exclusive events	305
9	Logic	256	10.5	Probability of combined events	307
9.1	Propositions	257	10.6	Tree diagrams and Venn diagrams	312
9.2	Compound statements	259	10.7	Probability 'with replacement' and 'without replacement'	318
9.3	Implication and equivalence	265	10.8	Conditional probability	322
9.4	Using truth tables	268			
9.5	Logical equivalence, tautology and contradiction	275			
9.6	Converse, inverse and contrapositive	279			

Topic 4: Statistical applications

333

11	The normal distribution	334	13	Chi-squared hypothesis testing	388
11.1	The normal distribution curve	334	13.1	The χ^2 statistic	388
11.2	Probability calculations using the normal distribution	339	13.2	The χ^2 test for independence	395
11.3	Solving practical problems with the normal distribution	345	13.3	Using your GDC to calculate the χ^2 statistic	398
11.4	Inverse normal calculations	350	13.4	Restrictions on using the χ^2 test	403
12	Correlation	364			
12.1	The concept of correlation	364			
12.2	Scatter diagrams	365			
12.3	Line of best fit	369			
12.4	Pearson's product moment correlation coefficient, r	371			
12.5	Regression line of y on x	376			
12.6	Using the equation of the regression line	380			

Topic 5: Geometry and trigonometry

409

14 Equation of a line in two dimensions	410	16 Geometry of three-dimensional solids	467
14.1 The gradient of a line	411	16.1 Finding the length of a line within a three-dimensional solid	469
14.2 The y-intercept	416	16.2 Finding the size of an angle in a three-dimensional solid	474
14.3 Finding the equation of a straight line	417	16.3 Calculating volumes and surface areas of three-dimensional solids	480
14.4 The equation of a straight line	420		
14.5 Drawing a straight line graph from an equation	425		
15 Trigonometry	435		
15.1 Trigonometric ratios	436		
15.2 Angles of elevation and depression	440		
15.3 Harder trigonometry problems	443		
15.4 The sine rule	446		
15.5 The cosine rule	450		
15.6 Area of a triangle	454		
15.7 Constructing labelled diagrams	457		

Topic 6: Mathematical models

492

17 Functions and graphs	493	19 Exponential and polynomial functions	545
17.1 What is a function?	493	19.1 Exponential functions and their graphs	545
17.2 Functions in more detail	496	19.2 Exponential models	552
17.3 Rational functions	504	19.3 Polynomial functions	558
17.4 Drawing graphs and diagrams	508	19.4 Modelling with polynomial functions	562
18 Linear and quadratic models	521		
18.1 Linear models	521		
18.2 Quadratic functions and their graphs	528		
18.3 Quadratic models	536		

Topic 7: Introduction to differential calculus

571

20 Introduction to differential calculus	572	21 Stationary points and optimisation	613
20.1 The derivative	573	21.1 Increasing and decreasing functions	613
20.2 Differentiation	577	21.2 Stationary points, maxima and minima	617
20.3 Rates of change	589	21.3 Optimisation	626
20.4 The second derivative	592		
20.5 Gradient of a curve at a given point	594		
20.6 Equation of the tangent at a given point	600		
20.7 Equation of the normal at a given point	605		
22 How to use your graphical calculator			640
22.1 Introduction			640
22.2 Getting started			641
22.3 GDC support by chapter			649
23 Internal assessment			692
23.1 Why is there a project in this course?			692
23.2 How do I start?			692
23.3 Is my topic a good one?			693
23.4 What is the structure of the project?			693
23.5 What are the assessment criteria?			694
23.6 What do I do when I have finished my project?			697
Answers			698
Glossary			749
Index			754

Sample examination papers

Two example examination papers written by the authors to support revision of this course are available online: visit education.cambridge.org/mathsstudiespaper1 and education.cambridge.org/mathsstudiespaper2

Introduction

This book has been written for students and teachers taking the Mathematical Studies SL course of the International Baccalaureate®. The syllabus is divided into seven topics and the book follows this structure, with each topic divided into chapters within that theme. The syllabus is designed to emphasise the practical mathematics that will enable students to continue to use their learning at university, in training courses, or when working, and the book encourages this. Explanations, worked examples and exercises are all firmly rooted in ideas that might be met in the course of normal experience.

As this book will be used by students from widely differing mathematical backgrounds, we have included fast forward and rewind indicators that mean it is not necessary to follow the order of the book exactly as it has been written; teachers will be able to integrate appropriate sections of the book with their own schemes of work:



The rewind indicators will refer the reader back to another section in the book if any prerequisite information is required to complete a section.



The fast forward indicators inform the reader where topics will be covered in more detail later in the book.

The concepts in Topic 6, Mathematical models, have been extended throughout the text. Problem solving is a vital skill, and confidence with this is achieved by students who realise that they can approach solutions from different perspectives. Worked examples aim to show that there might be several ways to find an answer: through drawing diagrams, working from first principles, through algebra or the use of technology.

It is important to understand that the Graphical Display Calculator (GDC) is fundamental to the course, and is allowed not only in both examinations, but also when students are working on their project. Where appropriate, all examples requiring the use of the GDC are illustrated by screen shots, and if students need additional support, the GDC chapter at the end of the book gives more detailed instruction of how to use their GDC. The calculators used in the writing of this book are the Casio *fx-9750G11* and the Texas TI-84 Plus Silver Edition. While many students might find they need to use slightly different key strokes from those that are given, the examples are close to those in the authors' experience of the many different versions of GDC that students have shown them.

How to use this book

Book structure

Each topic begins with an introduction that emphasises the place of that topic within mathematical history, practical experience, or with some information about internationally recognised mathematical people. It also lists any prior knowledge that is required, and any previous chapter that needs to be completed before this topic is started.

Each chapter of the book follows a pattern designed to be helpful to students from different national educational programmes, and familiar with different mathematical traditions. Like the topics, each chapter starts with an introduction that gives some context to the chapter content in terms of mathematical history, practical experience, or a historical person. There are also 'In this chapter you will learn' panels that list the learning objectives of the chapter.

The chapters contain numerous worked examples throughout. These are arranged with the formal written mathematics on the right-hand side (in blue font), and the thought processes or hints on the left-hand side in speech bubbles. Initially the worked examples can be followed using the ideas in the speech bubbles. For revision, the hints can be covered up, and the student can concentrate on the formal written mathematics that they will need in their examinations.

First, draw a diagram and label it with the information from the question.

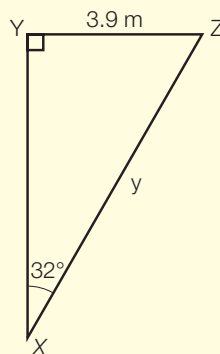
Then identify the ratio needed and solve the equation

To find XZ, you are using the opposite side and the hypotenuse. Choose SOH.

Using Pythagoras' Theorem

Worked example

Formal working, e.g.:



$$\text{a) } \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 32^\circ = \frac{3.9}{y}$$

$$y = \frac{3.9}{\sin 32^\circ}$$

$$y = 7.36 \text{ m}$$

$$\begin{aligned} \text{b) } XY^2 &= XZ^2 - YZ^2 \\ &= 7.36^2 - 3.9^2 \\ &= 6.24 \text{ m} \end{aligned}$$

At the end of each sub-section of a chapter is an exercise. This allows the student to check their knowledge and understanding of the chapter and the methods learned so far. These are in the format of drill-like questions that act as a quick self-assessment of progress.

Each chapter finishes with summary list of what the student should know once they have completed the chapter. This can be tested using the Mixed Examination practice questions that follow. These include a selection of exam-style questions written by the authors in the style of IB examination questions, and some real exam questions taken from past IB Mathematical Studies papers.

The questions are designed to make sure that students understand the mathematics studied in that chapter and how to use it. For some questions, particularly in the latter half of the book, students might be expected to use

concepts from more than one topic, which is a common theme for IB questions. It is very important that students have practised combining ideas and techniques before they attempt any past examination papers.

Supporting panels and icons

Within each chapter there are various support panels and icons to guide the student, or encourage them to look beyond their immediate studies.

Learning links

These are designed to link the course with more formal mathematical processes that might have been learned in the past. They **do not** have to be used because they are not on the syllabus. However, if the student wishes to see the connections between this syllabus and previous learning, the link is easy to follow.

Help with using the text

These are integrated throughout the book, often recalling the student's attention to a particular point in the text or a worked example, or reminding them of a general point of good mathematical practice.

These points allow students to easily go backwards or forwards to any section if they need reminders.

This book is based on a specific syllabus, but should not be seen solely as a guide to the final examinations. However, for all students it is reassuring to have common mistakes and misunderstandings pointed out, so that their examinations can be approached with confidence.

Icons linking in the wider IB Diploma ethos

The Theory of Knowledge is central to the IB Diploma. These panels aim to link questions met in the TOK course with ideas explored in the Mathematical Studies SL course.

These panels link to other subject areas providing cross-curricular support as well as links to wider applications of mathematics in the practical world.

Critical thinking is part of mathematics as it is a tool that can be used in many contexts and cultures. These panels highlight issues about the application of mathematics in the real world and to other subject areas. It also includes links to mathematical history, which places today's mathematics in context and shows the breadth and spread of mathematical ideas.

hint



exam tip



The internal assessment

All Mathematical Studies students complete a project as part of their diploma course. It is worth 20% of their final mark, and needs to be given approximately 20% of the time allowed for an SL subject.

Throughout the book, some comments are made about ideas and techniques that may be useful in projects. There is also a chapter at the end of the book giving advice in more depth, concentrating in particular on the different assessment criteria, giving advice about how to make sure that these are understood and correctly interpreted.



Use of technology

The use of technology is encouraged on this course. This does not need to be restricted to the use of the GDC. Personal computers (PCs, Mac, Laptop, tablets etc.) can also reinforce and encourage understanding; this can be through using commercial geometric or statistical packages, as well as programmes that students already know, such as spreadsheets. There are also many websites and videos online that can be very helpful; though students should be cautious about trusting the site before using them.

Online material

There are two practice examination papers available online at

education.cambridge.org/mathsstudiespaper1

education.cambridge.org/mathsstudiespaper2

Paper 1 has fifteen questions, and each question is marked out of six. The paper takes 90 minutes, so students should aim to gain one mark a minute.

Paper 2 has six questions, and the mark for each question can vary depending on its length and complexity. The total time for the paper is 90 minutes, and the total marks are also 90, as in Paper 1. In paper 2, if a question is marked out of 15, the student should aim to complete it in fifteen minutes.

We hope you find the Mathematical Studies SL course both interesting and accessible. Mathematics is a unique combination of understanding and practice, and we have written the text and exercises to help you with both sides of the subject. If you can build on the knowledge you already have, you should be able to move forward with a useful background in mathematics that will be relevant in future studies and your workplace.

Kwame Dwamena

Caroline Meyrick

Acknowledgements

The authors and publishers are grateful for the permissions granted to reproduce materials in either the original or adapted form. While every effort has been made, it has not always been possible to identify the sources of all the materials used, or to trace all copyright holders. If any omissions are brought to our notice, we will be happy to include the appropriate acknowledgements on reprinting.

IB exam questions © International Baccalaureate Organization. We gratefully acknowledge permission to reproduce International Baccalaureate Organization intellectual property. This work has been developed independently from and is not endorsed by the International Baccalaureate (IB).

TI-83/TI-84 font material reproduced with permission of Texas Instruments Incorporated.

TI makes no warranty regarding the accuracy of the information presented in the respective materials, including the fitness for use for your intended purpose. Also, TI is under no obligation to update the information presented in these materials.

Casio fonts are reproduced with permission of Casio Electronics Company Ltd (to access downloadable Casio resources go to <http://edu.casio.com/dl/>).

Text


p2 Ada Lovelace quotation: 'Note G' from Ada Lovelace's translation and commentary of a 1842 document called Sketch of the Analytical Engine by L.F. Menabrea (information sourced from www.allonrobots.com/ada-lovelace.html); p194 Data: the consumption of softwood throughout the UK for 2000-2009 www.forestry.gov.uk/forestry, p195 Data: average pump price of fuel per litre in 194 countries in 2010 www.data.worldbank.org; p277 U.S. field production of crude oil between 1920 and 2010 U.S. Energy Information Administration (2012); p592 Extract from Simon Singh's book *Fermat's Last Theorem* Simon Singh, *Fermat's Last Theorem* (London: Fourth Estate, 1997); p592 Quote of a saying of mathematician Hugo Rossi Hugo Rossi, *Notices of the American Mathematical Society*, (Vol 43, Number 10, 1996); p613 Quote from Ian Stewart's book *Seventeen Equations that Changed the World* Ian Stewart, *Seventeen Equations that Changed the World* (Profile Books, 2012); p232 1899 'Everything that can be invented has been invented' A misquote attributed to Charles Holland Duell, Commissioner of the US Patent Office, 1899; p232 1943 'only be a world market for maybe five computers' Commonly attributed to Thomas J. Watson, Chairman of International Business Machines (IBM), 1943; p232 Quote from Richard Brealey and Stewart Myers, *Principles of Corporate Finance* 1988 Richard Brealey and Stewart Myers, *Principles of Corporate Finance* (McGraw Hill, 1988).

Images

We would like to thank the following for permission to reproduce images:

Cover: © sharonzilberczvig / Alamy

p1 mbortolino / iStock; p2 Tim Jenner / Shutterstock; p9 Wejp-Olsen, Werner / CartoonStock; p20 London Community Gospel Choir; p37 maggio07 / iStock; p46 GeorgiasKollidas / Shutterstock; p50 carterdayne / iStock; p54 PinkBadger / iStock; p65 Phillip Lange / Shutterstock; p88 Nicku / Shutterstock; p98 vblinov / Shutterstock; p110 ene / Shutterstock; p110 Bjorn Hoglund / Shutterstock; p112 X-rates.com; p116 Aaron Bacall / CartoonStock; p128 courtesy of Nissan Motors GB Ltd; p141 DamirK / iStock; p182 Marty Bucella / Cartoonstock; p183 <http://hdl.loc.gov/loc.pnp/cph.3b11632> / Library of Congress; p206 bikeriderlondon / Shutterstock; p220 jenny / Shutterstock; p222 visuelldesign / Shutterstock; p232 Brocreative / Shutterstock;

A decorative border on the left side of the page features a vertical strip of numbers (0-9) in a light teal color. At the top left, there is a magnifying glass icon with a teal handle and lens, positioned over a small cluster of numbers. The background of the page is white with a faint, repeating pattern of numbers in the same teal color.

p233 © INTERFOTO / Alamy ; p233 mathisworks / iStock; p256 Nelson Minar /Sydney Powerhouse Museum; p265 S Harris / CartoonStock; p288 mikhailshiyarov / Shutterstock; p290 Arno van Dulmen /Shutterstock; p318 Pgiarn / iStock; p333tLledo / Shutterstock; p333bhttp://commons.wikimedia.org/wiki/File:Cancer_smoking_lung_cancer_correlation_from_NIH.png; p334 Bill Greenhead-Stik / CartoonStock; p335 bilwissedition Ltd. & Co. KG / Alamy; p371 SPL; p380 Christoff / Shutterstock; p395 dumayne /iStock; p409 Chinese Pythagorean theorem, from Joseph Needham's *Science and Civilization in China: Volume 3, Mathematics and the Sciences of the Heavens and the Earth*; p410 Pavila / Shutterstock; p440 Robert F Balazik /Shutterstock; p467 S Harris / CartoonStock; p477 Kotse /Shutterstock; p492 ardaguldogan /iStock; p524 Redshinestudio / Shutterstock; p536 s_oleg/Shutterstock; p539 sumak77 / iStock; p552 AKTS; p571 EpicStockMedia / iStock; p571 kolb_art / iStock; p572 Grafissimo / iStock; p572 Photos 12 / Alamy; p590 ZigaCamernik / Shutterstock; p596 McKevin / iStock; p613 The CAPN navigation software (www.thecapn.com); p626 RASimon / iStock.