

# Answers

Answers are given to 3 s.f. unless the question specifies otherwise.

## Chapter 1

### Exercise 1.1

- T
  - T
  - F;  $\sqrt{17}$  is irrational
  - F; 1.51 can be written as  $\frac{151}{100}$
  - T
  - F; all irrational numbers are real numbers
  - F; the sum of two integers is always an integer
  - T
- Answers may vary, for example:
  - 6
  - $\frac{25}{4}$
  - $\sqrt{5}$
  - 8
- 
- $-5.1, -2.5, -2, 0, \frac{6}{7}, \sqrt{2}, \sqrt{3}, 12$
  - 12, 0
  - 12, -2, 0
  - 12, -5.1, -2, 0,  $\frac{6}{7}, -2.5$
  - $\sqrt{2}, \sqrt{3}$ ; irrational numbers
- Natural
  - Natural, integer, rational
  - Negative integer
  - Irrational
  - Natural number

### Exercise 1.2

- 169 cm
  - 1 h 45 min
  - 3200
  - 220 AUD
  - $628 \text{ m}^2$
  - 75 kg
- 50
  - 200
  - 320
  - 2150
  - 20 460
- 300
  - 2000
  - 100
  - 67 900
  - 708 500
- 2
  - 32
  - 13
  - 113

### Exercise 1.3

- (i) 6.8      (ii) 6.82
  - (i) 153.8    (ii) 153.81
  - (i) 18.0      (ii) 17.97
  - (i) 0.2        (ii) 0.16
- 41.9
  - 45.6
  - 7.1
- $145.27 \text{ cm}^2$
  - $93.29 \text{ cm}^3$
  - $72.97 \text{ cm}^3$

### Exercise 1.4

- 93.5
  - 108
  - 0.00784
  - 8.56
  - 0.0626
- 3.08
  - 1080
  - 8.77
  - 6.69
- $43.5 \text{ cm}^2$
  - 3.18 cm
  - 8.60 cm
- $132 \text{ cm}^3$

### Exercise 1.5

- 72; 75.69
  - 60; 72.513...
  - 8; 7.34496...
- (i)  $138.18 \text{ cm}^2$ ;
  - (ii)  $138.2 \text{ cm}^2$ ;
  - (iii)  $138 \text{ cm}^2$
  - $3.18 \text{ cm}^2$
- $m = 30, n = 10, p = 100$  (to 1 s.f.)
  - 6.82
  - 5.573868149
  - 1.25
- $110\,304 \text{ m}^2$
  - When rounding either of the values to 1 s.f., he probably dropped a zero from the rounded value.

### Exercise 1.6

- (a) 2.65%  
(b) 7.32%  
(c) 6.11%  
(d) 31.8%
- (a)  $354.78 \text{ cm}^3$   
(b) 3.52%

### Exercise 1.7

- (a) 6 (b) 4 (c) -3  
(d) 6 (e) 0
- (a) 12 500 (b) 3080  
(c) 288 000 000 (d) 0.0421  
(e) 0.00972 (f) 0.00000838
- (a)  $6.21 \times 10^4$  (b)  $2.1 \times 10^3$   
(c)  $9.84 \times 10^7$  (d)  $5.2 \times 10^1$
- (a)  $7.27 \times 10^{-1}$  (b)  $3.19 \times 10^{-2}$   
(c)  $2.57 \times 10^{-6}$  (d)  $4.08 \times 10^{-4}$
- (a)  $398 \times 10^1$ ,  $0.17 \times 10^3$ ,  
 $370 \times 10^2$ ,  $0.02 \times 10^2$   
(b)  $3.98 \times 10^3$ ,  $1.7 \times 10^2$ ,  
 $3.70 \times 10^4$ ,  $2 \times 10^0$   
(c)  $3.8 \times 10^{-5}$ ,  $2.4 \times 10^{-3}$ ,  
 $2 \times 10^0$ ,  $1.2 \times 10^2$ ,  $1.7 \times 10^2$ ,  
 $3.98 \times 10^3$ ,  $3.70 \times 10^4$
- (a)  $3.17 \times 10^{10}$  (b)  $9.89 \times 10^{-2}$   
(c)  $4.56 \times 10^{-9}$  (d)  $1.54 \times 10^{-6}$   
(e)  $8.12 \times 10^4$  (f)  $3.44 \times 10^{-7}$
- (a)  $3.90 \times 10^3$  (b)  $3.90 \times 10^3$   
(c)  $2.65 \times 10^0$  (d)  $5.74 \times 10^6$
- (a) 33 min  
(b) 12 min  
(c) 79 min
- 1270 times (to 3 s.f.)

### Exercise 1.8

- (a) 395 s  
(b) 9 min 22 s  
(c) 3 d 6 h  
(d) 1 h 48 min 20 s  
(e) 17 595 min  
(f) 22 030 s

### Exercise 1.9

1.

City	Miami	Riga	Milan	Bahrain	Lima	Perth	Moscow
Celsius ( $^{\circ}\text{C}$ )	27.8	-2	7	18.3	25	32.2	-12
Fahrenheit ( $^{\circ}\text{F}$ )	82	28.4	44.6	65	77	90	10.4

### Exercise 1.10

- (a)  $38 \text{ km h}^{-1}$   
(b) 7.19 h  
(c) 29.3 km
- (a)  $2.625 \text{ km h}^{-1}$   
(b)  $2.64 \text{ km h}^{-1}$ ; faster  
(c)  $2.63 \text{ km h}^{-1}$

### Exercise 1.11

- (a) 3.5 m (b) 2760 mm  
(c) 4.8 km (d) 35 200 cm
- (a) 5800 g  
(b) 0.03 kg  
(c) 1.26 g  
(d) 1 000 000 mg
- (a)  $45 000 \text{ cm}^2$   
(b)  $0.0685 \text{ m}^2$   
(c)  $1.4 \times 10^6 \text{ m}^2$   
(d)  $1.2 \text{ cm}^2$
- (a)  $1.2 \times 10^7 \text{ cm}^3$   
(b)  $0.024 \text{ m}^3$   
(c)  $1300 \text{ mm}^3$   
(d)  $5 \times 10^5 \text{ cm}^3$
- (a) 7.91 (b) 3950 ml  
(c)  $83 300 \text{ cm}^3$  (d)  $687 \text{ cm}^3$
- (a) (i)  $70 000 \text{ cm}^2$   
(ii)  $7 \text{ m}^2$   
(b) (i)  $8.21 \times 10^5 \text{ cm}^3$   
(ii)  $0.821 \text{ m}^3$   
(c)  $33.1 \text{ cm}^2$
- (a) 38 (b) 16 cm
- 1250
- (a) 53 (b)  $20 \text{ cm}^3$
- (a)  $255 \text{ m}^2$  (b)  $20.4 \text{ m}^3$

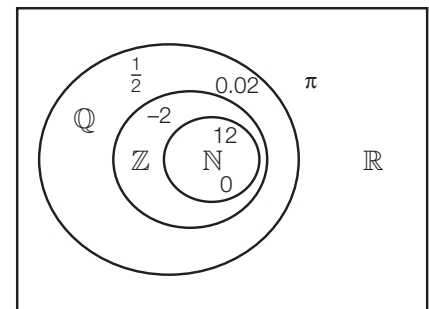
### Mixed examination practice 1

#### Exam-style questions 1

1.

	11	$\frac{1}{11}$	$\sqrt{11}$	-11
N	✓			
Z	✓			✓
Q	✓	✓		✓
R	✓	✓	✓	✓

2.



- (a) 2, natural number  
(b)  $4\frac{8}{9} = 4.8$ , rational number  
(c) 2, natural number  
(d) 42.7256..., irrational number  
(e) 10, natural number  
(f) -36, integer  
(g)  $-\frac{101}{3}$ , rational number  
(h) -4, integer
- (a)  $40.0 \text{ cm}^2$  (b)  $8.4 \text{ cm}^2$   
(c) 6.4 m
- (a)  $R = 20$ ,  $r = 10$ ,  $\pi = 3$  (1 s.f.)  
(b)  $A \approx 900$   
(c) 788

6. (i) 2.7 (ii) 2.66
7.  $9.09 \times 10^6$
8. (a)  $958 \text{ kg m}^{-3}$  (b)  $10^6 \text{ cm}^3$   
(c) 9.58 g
9. (a) 11 305 kg  
(b)  $4.42 \times 10^{-6} \text{ m}^3 = 4.42 \text{ cm}^3$

### Past paper questions 1

1. (a)  $1.265 \times 10^{-1}$   
(b) 0.13  
(c) 2.77%
2. (a) 144.75  
(b)  $1.4475 \times 10^2$   
(c) (i)  $96 \text{ m}^2$  (ii) 6.67%

## Chapter 2

### Exercise 2.1

1. (a), (d), (e) and (f) are linear;  
(b) and (c) are not
2. B and D

### Exercise 2.2

1. (a)  $y = x + 4$   
(b)  $y = -3x + 7$   
(c)  $y = \frac{1}{9}x - \frac{15}{9}$   
(d)  $y = -\frac{5}{2}x + \frac{15}{2}$   
(e)  $y = x - \frac{10}{11}$   
(f)  $y = \frac{1}{3}x - \frac{13}{6}$   
(g)  $y = -\frac{3}{4}x + \frac{27}{2}$   
(h)  $y = \frac{15}{4}x + 1$   
(i)  $y = \frac{2}{9}x + 7$
2. (a)  $5x - y + 4 = 0$   
(b)  $x - 2y - 5 = 0$   
(c)  $3x + 2y - 6 = 0$   
(d)  $3x - 4y - 10 = 0$   
(e)  $9x - 2y = 0$   
(f)  $5x + 2y - 1 = 0$

### Exercise 2.3

1. (a)  $m = 1$   
(b)  $z = 70$   
(c)  $y = -1$   
(d)  $x = 16$
2. (a)  $m = \frac{10}{3}$   
(b)  $f = 0.2$   
(c)  $x = 2.8$
3. (a)  $x = \frac{1}{8}$   
(b)  $z = 4.25$   
(c)  $y = 19$
4. (a)  $x = 13$   
(b)  $y = 5$   
(c)  $m = 2$   
(d)  $x = -\frac{1}{16}$

### Exercise 2.4

1. (a)  $x = 8, y = 13$   
(b)  $x = 3, y = 4$   
(c)  $x = 0.5, y = 4$   
(d)  $x = 9, y = 8$   
(e)  $x = -0.538, y = -2.77$   
(f)  $x = 2.81, y = 0.484$   
(g)  $x = 3.6, y = 1.6$   
(h)  $x = 0.4, y = 2.47$
2. (a)  $s = 5.14, t = -3.79$   
(b)  $s = 2.83, t = -1.16$   
(c)  $s = -1.60, t = -3.49$   
(d)  $s = 197, t = 33.3$

### Exercise 2.5

1. (a)  $3m + b = 85$   
(b)  $m = 22, b = 19$
2. (a)  $6s + 2t = 100$   
(b) Snickers: 6 AED; Twix: 32 AED
3.  $x + y = 97$  and  $x - y = 23$ ; 60 and 37
4. (a)  $2c + 7d = 128.91$   
(b) CDs: £11.99; DVDs: £14.99
5. Batteries: £2.99; calculators: £14.50

6. (a)  $3x + 4y = 2987, 2x + 5y = 3123$  (where  $x$  is the price of an easy-click laptop and  $y$  is the price of a smooth-tab laptop)  
(b) Easy-click: \$349; smooth-tab: \$485
7. \$345
8. 9 shorter and 6 longer questions
9. (a)  $S(-\frac{7}{3}, 0)$ , and  $T(0, 7)$   
(b)  $R(-2, 1)$

### Exercise 2.6

1. (a), (b), (d), (f), (h) and (i)
2. A and D
3. (a) Max (b) Min  
(c) Min (d) Max
4. (a)  $x^2 + x = 0$   
(b)  $x^2 - 2x - 3 = 0$   
(c)  $4x^2 + x - 4 = 0$   
(d)  $x^2 - 5x - 6 = 0$   
(e)  $x^2 + 5x - 15 = 0$   
(f)  $x^2 - 6x - 7 = 0$

### Exercise 2.7

1. (a) 1, -1.33  
(b) 2, 0.571  
(c) -2.5, 0.667  
(d) -1.12, 10.7  
(e) 14.8, 0.203  
(f) -0.805, 1.74
2. (a)  $2x^2 + 3x - 2 = 0$ ; 0.5, -2  
(b)  $3x^2 + 11x - 9 = 0$ ; 0.689, -4.36  
(c)  $x^2 + 7x - 13 = 0$ ; 1.52, -8.52  
(d)  $6x^2 + 7x - 3 = 0$ ; 0.333, -1.5  
(e)  $4x^2 - 5x - 8 = 0$ ; 2.17, -0.921  
(f)  $9x^2 + x - 4 = 0$ ; -0.724, 0.613

3. (a) 2.70, -1.16  
 (b) 0.614, -2.24  
 (c) 4.11, -0.608  
 (d) 3.95, 0.198  
 (e) 0.414, -2.41  
 (f) 1.26, -1.59

### Exercise 2.8

1.  $n^2 + n - 306 = 0$ ; 17 and 18
2. (a)  $x^2 + 7x - 60 = 0$   
 (b) 5 cm and 12 cm
3. (a)  $x^2 + x - 156 = 0$   
 (b)  $x = 12$   
 (c) Parallel sides 12 cm and 18 cm, height 10 cm
4. (a) 1.24 s and 2.97 s  
 (b) 5.46 s

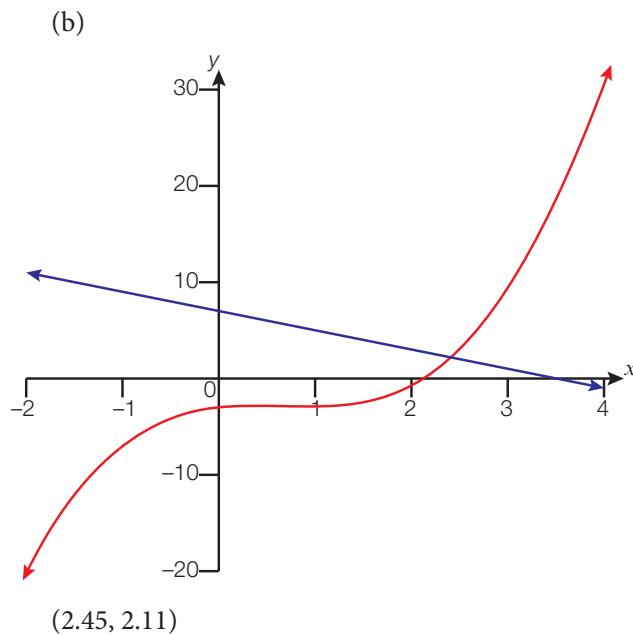
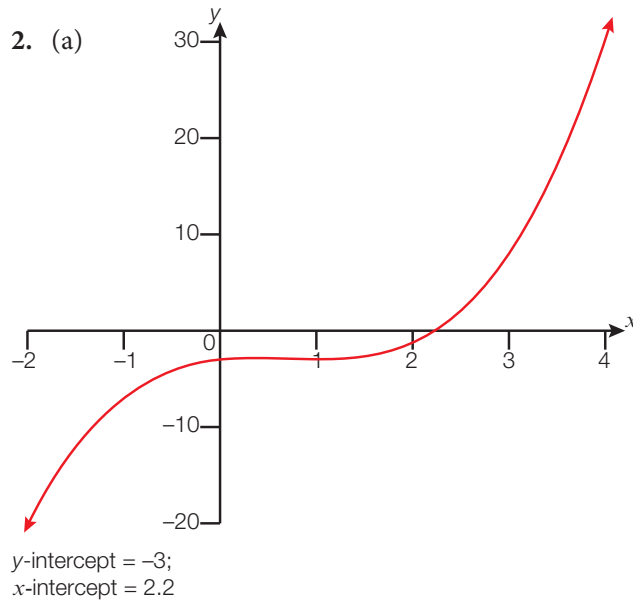
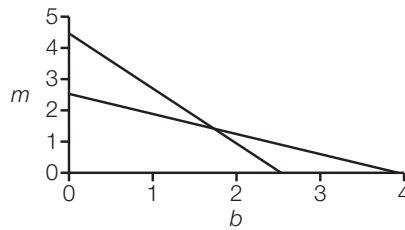
### Mixed examination practice 2

#### Exam-style questions 2

1.  $x = 6$
2. (a)  $x = -1, y = 2$   
 (b)  $x = 1.24, y = 2.33$   
 (c)  $x = -2.91, y = 1.57$
3. (a) 0.286, 0.333  
 (b) 4.85, -1.85  
 (c) 4.92, -6.92  
 (d) 8.72, -1.72  
 (e) 1.61, -0.811
4. (a)  $4x + 3y = 5529, 2x + 5y = 6751$   
 (b) Bracelet: 528 INR;  
 pendant: 1139 INR
5. (a)  $m + c = -5, 4m + c = 4$   
 (b)  $m = 3, c = -8$   
 (c) No
6. (a)  $a + 820b = 106.24,$   
 $a + 650b = 85.84$   
 (b)  $a = 7.84, b = 0.12$   
 (c) £97.24
7. 20 m by 38 m
8. (a) 1.22 s, 3.56 s  
 (b) 4.78 s

### Past paper questions 2

1. (a)  $6b + 9m = 23.40$   
 (b)  $b = 1.80, m = 1.40$   
 (c)



## Chapter 3

### Exercise 3.1

- (c), (d) and (f)
- (a) 3; 14, 17, 20  
(b) -3; -7, -10, -13  
(c) -33; 251, 218, 185  
(d) 21; 252, 273, 294  
(e) 4.2; 41.3, 45.5, 49.7  
(f)  $\frac{3}{4}$ ;  $\frac{11}{4}$ ,  $\frac{7}{2}$ ,  $\frac{17}{4}$   
(g)  $\frac{2}{3}$ ;  $\frac{18}{7}$ ,  $\frac{68}{21}$ ,  $\frac{82}{21}$   
(h)  $-x - 9$ ;  $-x - 20$ ,  $-2x - 29$ ,  $-3x - 38$

### Exercise 3.2

- (a) 115, 163 (b) 435, 855  
(c) 58, -124 (d) 142, 340  
(e) 105, 337 (f) 970.8, 768  
(g) -372, -820 (h) 428.4, 723.6  
(i)  $\frac{134}{35}$ ,  $\frac{197}{35}$
- (a) 20, 32 (b) 149, 226  
(c) 67, 191 (d) -88, -116  
(e) -88, -250 (f) -267, -998  
(g) 3.51, 6.4 (h)  $\frac{37}{18}$ ,  $\frac{50}{9}$   
(i)  $17x + 62$ ,  $24x + 97$

### Exercise 3.3

- (a) 3 (b) 8  
(c) 11 (d) -2.3  
(e) -13.65 (f) 8  
(g) 26 (h) -2.1  
(i) -1.9 (j) 0.563
- (a) 36 (b) 27  
(c) 46 (d) 39  
(e) 55 (f) 100  
(g) 46 (h) 75  
(i) 46 (j) 58
- (a) 12 (b) 19  
(c) 400 (d) 198  
(e) 56 (f) 188  
(g) 1990 (h) 88  
(i) 205 (j) 60

- (a)  $u_1 + 4d = 9$ ,  $u_1 + 10d = 45$   
(b)  $u_1 = -15$ ,  $d = 6$   
(c) 279
- (a)  $u_1 = 64$ ,  $d = 18$   
(b) 406
- (a)  $u_1 = 172$ ,  $d = -17$   
(b) -440
- (a)  $d = -6$ ,  $u_1 = -34.93$   
(b) -268.93  
(c) No

### Exercise 3.4

- (a) 47  
(b) 13 months
- (a) 100  
(b) In the 20th week
- (a) 97 minutes  
(b) After the 19th week  
(c) 161 minutes
- (a) 132 tonnes  
(b) In the 16th year
- (a) 800 rupees  
(b) In the 36th month

### Exercise 3.5

- (a) 710 (b) 6576  
(c) -696 (d) -2300  
(e) 3259.2 (f) -3438
- (a) 32 (b) 24  
(c) 28 (d) 24  
(e) 80 (f) 310

### Exercise 3.6

- (a) 500 (b) 3042  
(c) 5565 (d) -3010  
(e) 50 (f) 500  
(g) -1305 (h) -3565.65
- (a) 558 (b) 2270  
(c) 6006 (d) -14840  
(e) -1260 (f) 1372.5  
(g) -341.25 (h) 794.592

### Exercise 3.7

- (a) 2376 (b) 2168  
(c) -11940 (d) 2207.52  
(e)  $\frac{129}{4} = 32.25$
- (a) 20, 2750 (b) 32, 17136  
(c) 19, 363.85 (d) 30, -4605  
(e) 31, 1666.25
- (a) 3240 (b) 14630  
(c) 14850 (d) 4215  
(e) 16830
- (a) 97 (b) 17  
(c) 17140
- (a) 1700  
(b)  
 $S_n = \frac{n}{2}(2u_1 + (n-1)d)$   
i.e.  $5800 = \frac{n}{2}(2 \times 28 + 6(n-1))$   
 $= 28n + 3n(n-1)$   
 $= 25n + 3n^2$   
(c) 40

### Exercise 3.8

- (a) €1000 (b) €5950  
(c) €12,650
- (a)  $u_{10} = u_1 + (n-1)d$   
 $= 3 + (10-1) \times 2$   
(b) 440  
(c) 9th week
- (a) \$103 (b) \$10,816
- (a) 40,500 nairas  
(b) 169,500 nairas

### Exercise 3.9

- (b), (d), (e), (g) and (h)
- (a) 32, 64, 128  
(b) 43.2, 25.92, 15.552  
(c) 27, 9, 3  
(d) 52.704, 63.2448, 75.89376
- (a) 270, 151.875  
(b) 3.125, 6.25  
(c) -18, -54  
(d)  $\frac{1}{25}$ ,  $\frac{1}{125}$ ,  $\frac{1}{625}$

### Exercise 3.10

- (a)  $15/10 = 22.5/15 = 1.5$   
(b)  $u_1 = 10, r = 1.5$   
(c) 384.4
- (a)  $r = 2, u_1 = 0.75, u_{10} = 384$   
(b)  $r = 3, u_1 = \frac{1}{3}, u_{10} = 6561$
- (a)  $u_1 = 2.5$  and  $r = 2$ , or  
 $u_1 = -2.5$  and  $r = -2$   
(b) 5120
- (a) 4, 786 432  
(b) 1.5,  $1.42 \times 10^5$  (3 s.f.)  
(c) 3.2,  $3.48 \times 10^{10}$  (3 s.f.)  
(d) 1.5, 438 (3 s.f.)  
(e) -1.1, -514 (3 s.f.)  
(f) 2.2,  $-3.17 \times 10^8$  (3 s.f.)
- (a) 1.5; 4.5, 6.75, 10.125  
(b) -2; -4, 8, -16  
(c) 0.5; 25, 12.5, 6.25  
(d) 1.1; 1.21, 1.331, 1.4641

### Exercise 3.11

- (a) 9 (b) 4  
(c) 6 (d) 8  
(e) 6 (f) 7
- (a) 5 (b) 7  
(c) 11 (d) 11  
(e) 13 (f) 11
- 5
- (a) 12 (b) 8  
(c) 7 (d) 8  
(e) 8 (f) 6

### Exercise 3.12

- (a) 2046  
(b) 29 296.8  
(c)  $4.83 \times 10^8$  (3 s.f.)  
(d) 54 613.125  
(e) 39.6 (3 s.f.)  
(f)  $1.32 \times 10^7$  (3 s.f.)
- (a) 625  
(b) 1600  
(c) 2.28 (3 s.f.)  
(d) 56.0 (3 s.f.)  
(e) 47.4 (3 s.f.)  
(f) 808 (3 s.f.)

- (a) 4092  
(b) 47300 (3 s.f.)  
(c) -712 (3 s.f.)  
(d)  $1.96 \times 10^{10}$  (3 s.f.)  
(e) 5620 (3 s.f.)
- (a) 2; 10 485 750  
(b) 3;  $4.24 \times 10^{11}$  (3 s.f.)  
(c) 0.5; 256 (3 s.f.)  
(d) 1.2; 350 (3 s.f.)  
(e) 0.2; 7.81 (3 s.f.)  
(f) 2; 8 388 600  
(g) 343;  $7.05 \times 10^{60}$  (3 s.f.)
- (a) 0.4  
(b) 1500 (3 s.f.)
- (a)  $u_1 = 1000, r = 0.5$   
(b) 2000 (3 s.f.)
- 63.996
- (a)  $u_1 = 5, r = 3$   
(b) 107 616 800  
(c) 108 000 000 (3 s.f.)  
(d)  $1.08 \times 10^8$  (3 s.f.)

### Exercise 3.13

- 9430 (3 s.f.)
- 124,000 AUD
- (a) 9353.60, 9540.88, 9731.91,  
9926.77, 10125.53,  
10328.26, 10535.06,  
10745.99  
(b) £98,448.00
- (a) £817.59 (b) £4457.36

### Mixed examination practice 3

#### Exam-style questions 3

- (a) 17 (b) 347 (c) 3710
- (a)  $u_1 + 4d = 42, u_1 + 8d = 64$   
(b)  $u_1 = 20, d = 5.5$
- (a) 51 (b) 1272  
(c) 576 (d) 15
- (a) 0.8 (b) 1960 (3 s.f.)
- 0.754 m (3 s.f.)
- (b) €145.86, €186.16  
(c) €1509.35

- (b) 7.32 h (3 s.f.)  
(c) 107 h (3 s.f.)
- Option 2
- (i) (a) \$305  
(b) \$13,050  
(ii) (a) \$16,560  
(b) 51st month
- (a) 326 000 bricks (3 s.f.)  
(b) 2 600 000 bricks (3 s.f.)

### Past paper questions 3

- (b) 11, 18, 25 (c) 7  
(d) 144 (e) 900
- (a) 10  
(b)  $\frac{1}{3}$   
(c) 1.50 (3 s.f.)  
(d) Both  $(\frac{1}{3})^{10}$  and  $(\frac{1}{3})^{1000}$   
(or those numbers divided  
by  $\frac{2}{3}$ ) are 0 when corrected  
to 3 s.f.  
(e) 29 525.5
- (a) 1140  
(b)  $6r^5 = 16 \times 12$   
(c) 2
- (a) (i) \$2050 (ii) \$5120  
(b) \$11,500  
(c) Total value after 10 weeks:  
option one \$10,000; option  
two \$11,500; option three  
\$10,230. Therefore, option  
two would be best.

## Chapter 4

### Exercise 4.1

- (a) 1175.88 CAD  
(b) 967.74 GBP  
(c) 805.30 AUD  
(d) 902.00 EUR  
(e) 3201.07 USD
- (a) 1446 CAD  
(b) \$561  
(c) ¥46 556
- (a) €728.79  
(b) 152.15 CHF

### Exercise 4.2

- (a) 37,490.88 THB  
(b) 3.55 SGD
- (a) £147 (b) €171.99  
(c) \$8.21
- (a) €653.36 (b) \$861.24  
(c) \$38.76
- (a) £1005.43 (b) £1004.83  
(c) Bank A
- (a) 2842.51 HKD (b) £63.02

### Exercise 4.3

- (a) €53,529.02  
(b) €53,874.20  
(c) €53,954.01
- (a) 520,302.00 AUD  
(b) 520,370.77 AUD  
(c) 520,397.39 AUD
- 5.64% (3 s.f.)
- 15.7% (3 s.f.)
- (a) 16.8 years (b) 26.7
- (a) £76,379.84  
(b) No; investment is worth  
£91,154.38 after 10 years.

### Exercise 4.4

- (a) £64 (b) £46,620
- (a) £832 (b) £49,920  
(c) £592  
(d) Arthur; Ken repaid a total  
of £53,280, which is greater  
than Arthur's total by  
£3360.
- (a) £2500 (b) £22,500  
(c) £549 (d) £3852
- (a) €86.32 (b) €5179.20  
(c) €799.20  
(d) Yes; they would have saved  
about €296.
- (a) Monthly payments:  
£387.72; customer deposit  
£17,108  
(b) Total amount of credit:  
£19,404; total amount  
payable: £36,428

- (a) \$160.32 (b) \$9619.20  
(c) \$1619.20
- (a) \$56,346.36  
(b) \$16,843.36  
(c) No; option 3 would make  
them worse off by \$1165.44
- (a) ¥1234.82 (b) ¥44,453.52  
(c) ¥1453.52

### Exercise 4.5

- £433,842.38
- €54112.55
- ¥4.30 million (3 s.f.)

Commodity	Overall rate of inflation, %	Annual % rate of inflation
Rump steak, British	10.3	0.702
Cod fillets	50.5	2.96
Sugar, granulated	19.4	1.27
Cheese, Cheddar	71.8	3.94
Apples, eating	21.4	1.39
Carrots	-3.39	-0.246

- 20 years
- 10

### Exercise 4.6

- (a) £250.76 (b) £286.19
- (a) \$15,383.74  
(b) \$34,611.26  
(c) Yes; he would be better off  
by \$399.01.
- 2.84 (3 s.f.)
- 10.4 years (3 s.f.)
- (a) \$179,200  
(b) 32.0 (3 s.f.)  
(c) 3.06 years (3 s.f.)

### Mixed examination practice 4

#### Exam-style questions 4

- (a) 23,763.52 RUB  
(b) 235.66 CAD
- (a) \$10,463.23  
(b) \$3800.39
- (a) £12,957.15  
(b) 44.9% (3 s.f.)
- (a) 207,184.07 ZAR  
(b) 14.8 years (3 s.f.)  
(c) 23.4 (3 s.f.)
- (a) 25 months (b) \$184.73
- (a) 7.26  
(b) (i) \$34,768.38  
(ii) \$30,221.00
- (a) Monthly payment £700;  
(b) total interest £2151.76;  
(c) total amount payable  
£25,151.76
- (a) £19,071.50  
(b) 11.2%  
(c) £4062.10  
(d) 11.7%  
(e) Yes; if the annual rate of  
depreciation stayed at 14%  
(value based on salesman's  
claim), the car would be  
worth less than £20,000  
after 3 years.

### Past paper questions 4

- A: \$1200; B: \$1239.51; C: \$1230; D: \$1273.37
  - D; the total allowance is the highest (or grows the fastest).
  - 10%
- \$29,263.23
  - In the 4th year (3.46 years)
    - \$298.20
- \$2700, \$2900
    - \$6300
  - \$2160
    - 6% (d) \$3523.93
- €612.80 (b) \$780.64
  - \$19.36, 2.42%
- €16,857.45 (b) 30.6

## Chapter 5

### Exercise 5.1

- Continuous
  - Discrete
  - Continuous
  - Discrete
  - Discrete
  - Continuous
  - Continuous
  - Discrete
  - Discrete
  - Continuous

### Exercise 5.2

- | Mark | Frequency |
|------|-----------|
| 5    | 4         |
| 6    | 4         |
| 7    | 6         |
| 8    | 4         |
| 9    | 4         |
| 10   | 3         |

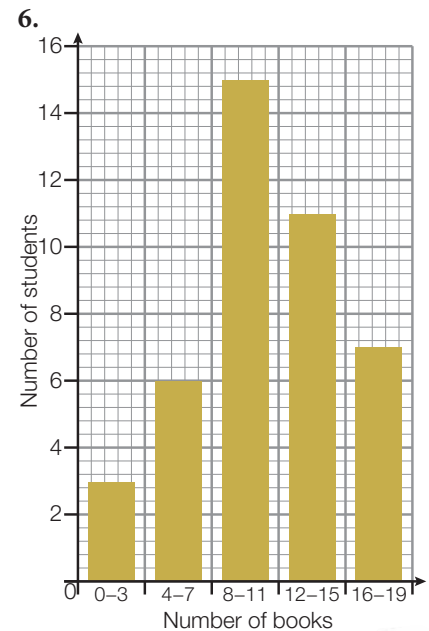
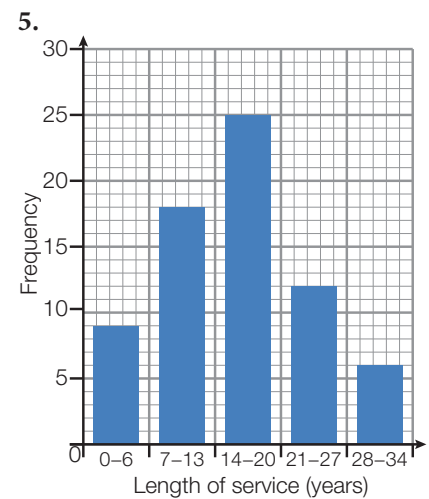
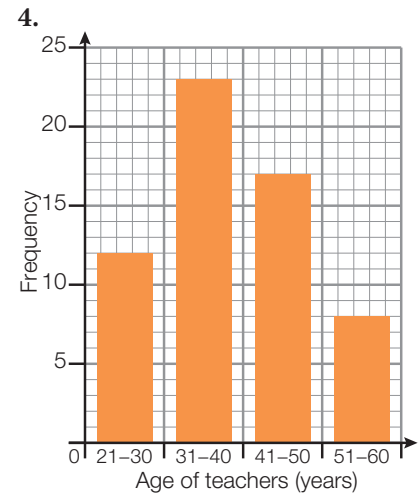
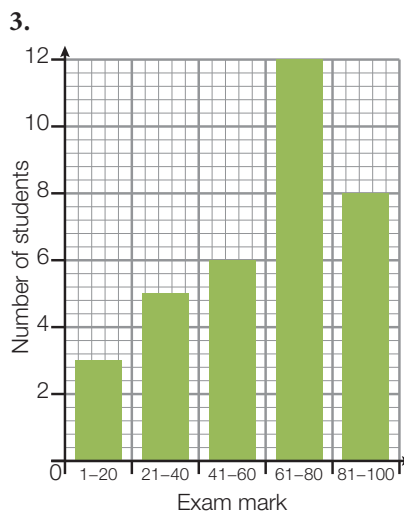
- | Number of rejects | Frequency |
|-------------------|-----------|
| 0                 | 10        |
| 1                 | 6         |
| 2                 | 7         |
| 3                 | 7         |
| 4                 | 4         |
| 5                 | 2         |

- | Home runs | Frequency |
|-----------|-----------|
| 23        | 1         |
| 24        | 2         |
| 25        | 1         |
| 26        | 2         |
| 27        | 4         |
| 28        | 4         |
| 29        | 0         |
| 30        | 0         |
| 31        | 2         |
| 32        | 0         |
| 33        | 0         |
| 34        | 1         |

### Exercise 5.3

- | Number of students | Frequency |
|--------------------|-----------|
| 0-4                | 4         |
| 5-9                | 6         |
| 10-14              | 16        |
| 15-19              | 7         |
| 20-24              | 7         |

- | Number of CDs | Frequency |
|---------------|-----------|
| 45-49         | 2         |
| 50-54         | 7         |
| 55-59         | 4         |
| 60-64         | 2         |
| 65-69         | 11        |
| 70-74         | 4         |





### Exercise 5.4

Answers may vary, depending on how the classes are defined in each case.

1.

Time (s)	Frequency
12–16	5
16–20	13
20–24	9
24–28	2
28–32	1

2.

Distance (m)	Frequency
5–7	8
7–9	8
9–11	7
11–13	5
13–15	2

3.

Distance (m)	Frequency
1.5–2.5	2
2.5–3.5	2
3.5–4.5	12
4.5–5.5	8
5.5–6.5	6

4.

Time (s)	Frequency
60–64	1
64–68	6
68–72	6
72–76	12
76–80	3
80–84	2

5.

Distance (m)	Frequency
25–30	5
30–35	9
35–40	9
40–45	5
45–50	2

### Exercise 5.5

1.

Time (s)	Class boundaries	Frequency	Class width	Mid-interval value
$18 \leq t < 20$	18–20	3	2	$(18 + 20) \div 2 = 19$
$20 \leq t < 22$	20–22	4	2	21
$22 \leq t < 24$	22–24	6	2	23
$24 \leq t < 26$	24–26	10	2	25
$26 \leq t < 28$	26–28	3	2	27
$28 \leq t < 30$	28–30	2	2	29

2. (Exercise 5.3 question 3)  
 Class boundaries: 1–20, 21–40, 41–60, 61–80, 81–100  
 Class widths: all 19  
 Mid-interval values: 10.5, 30.5, 50.5, 70.5, 90.5

(Exercise 5.3 question 4)  
 Class boundaries: 21–30, 31–40, 41–50, 51–60  
 Class widths: all 9  
 Mid-interval values: 25.5, 35.5, 45.5, 55.5

(Exercise 5.3 question 5)  
 Class boundaries: 0–6, 7–13, 14–20, 21–27, 28–35  
 Class widths: all 6  
 Mid-interval values: 3, 10, 17, 24, 31

(Exercise 5.3 question 6)  
 Class boundaries: 0–3, 4–7, 8–11, 12–15, 16–19  
 Class widths: all 3  
 Mid-interval values: 1.5, 5.5, 9.5, 13.5, 17.5

3. Answers may vary depending on how the classes were defined in Exercise 5.4.

(Exercise 5.4 question 1)  
 Class boundaries: 12–16, 16–20, 20–24, 24–28, 28–32  
 Class widths: 4  
 Mid-interval values: 14, 18, 22, 26, 30

(Exercise 5.4 question 2)  
 Class boundaries: 5–7, 7–9, 9–11, 11–13, 13–15  
 Class widths: 2  
 Mid-interval values: 6, 8, 10, 12, 14

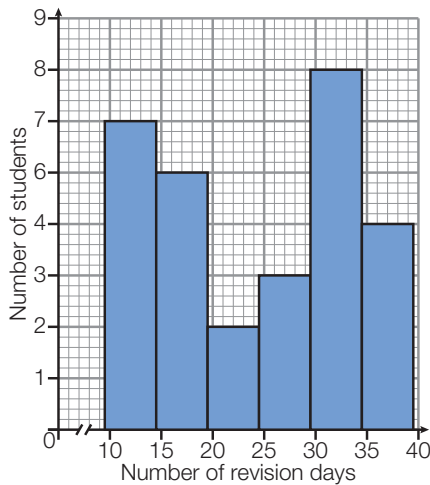
(Exercise 5.4 question 3)  
 Class boundaries: 1.5–2.5, 2.5–3.5, 3.5–4.5, 4.5–5.5, 5.5–6.5  
 Class widths: 1  
 Mid-interval values: 2, 3, 4, 5, 6

(Exercise 5.4 question 4)  
 Class boundaries: 60–64, 64–68, 68–72, 72–76, 76–80, 80–84  
 Class widths: 4  
 Mid-interval values: 62, 66, 70, 74, 78, 82

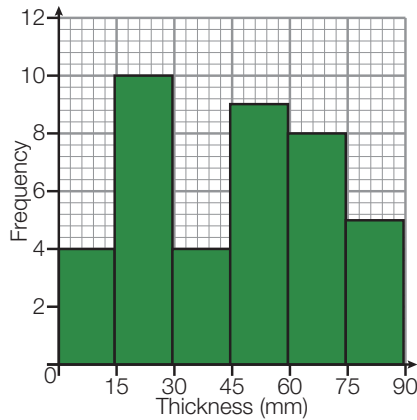
(Exercise 5.4 question 5)  
 Class boundaries: 25–30, 30–35, 35–40, 40–45, 45–50  
 Class widths: 5  
 Mid-interval values: 27.5, 32.5, 37.5, 42.5, 47.5

## Exercise 5.6

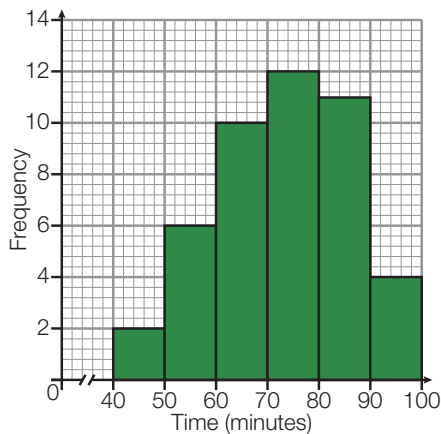
1. Frequencies: 7, 6, 2, 3, 8, 4



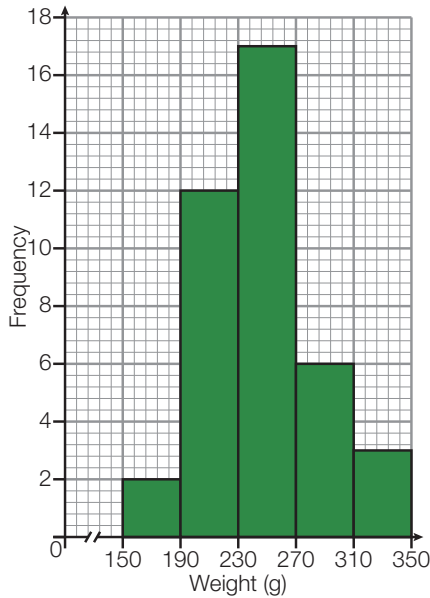
2. Frequencies: 4, 10, 4, 9, 8, 5



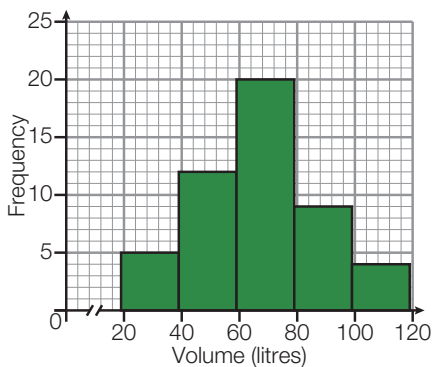
3. Frequencies: 2, 6, 10, 12, 11, 4



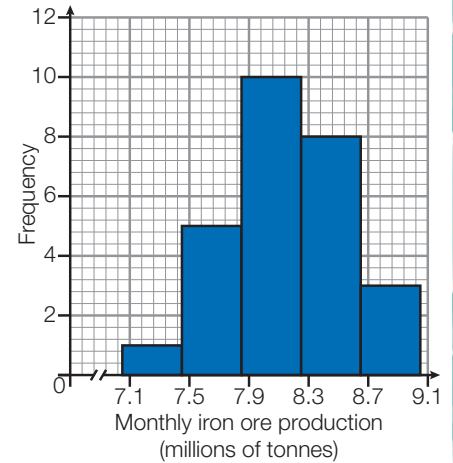
4. Class boundaries: 150.5–190.5, 190.5–230.5, 230.5–270.5, 270.5–310.5, 310.5–350.5  
Class widths: 40  
Mid-interval values: 170.5, 210.5, 250.5, 290.5, 330.5  
Frequencies: 2, 12, 17, 6, 3



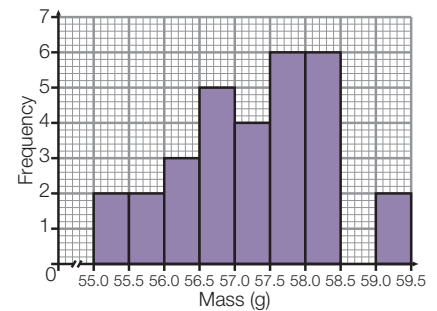
5. Class boundaries: 19.5–39.5, 39.5–59.5, 59.5–79.5, 79.5–99.5, 99.5–119.5  
Class widths: 20  
Mid-interval values: 29.5, 49.5, 69.5, 89.5, 109.5  
Frequencies: 5, 12, 20, 9, 4



6. Answers may vary depending on how the classes were defined.  
Class boundaries: 7.05–7.45, 7.45–7.85, 7.85–8.25, 8.25–8.65, 8.65–9.05  
Frequencies: 1, 5, 10, 8, 3



7. Class boundaries: 55.0–55.5, 55.5–56.0, 56.0–56.5, 56.5–57.0, 57.0–57.5, 57.5–58.0, 58.0–58.5, 58.5–59.0, 59.0–59.5  
Class widths: 0.5  
Mid-interval values: 55.25, 55.75, 56.25, 56.75, 57.25, 57.75, 58.25, 58.75, 59.25  
Frequencies: 2, 2, 3, 5, 4, 6, 6, 0, 2

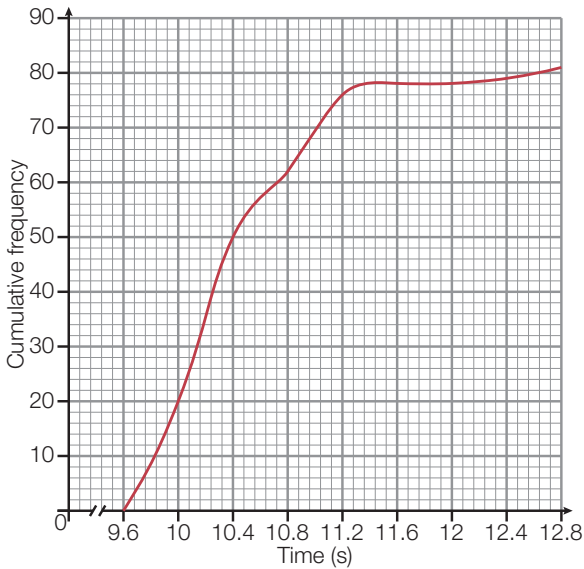


## Exercise 5.7

1. 5, 12, 17, 25, 33, 42, 45  
2. 1, 3, 8, 18, 30, 34, 37  
3. 2, 8, 17, 32, 42, 49, 52

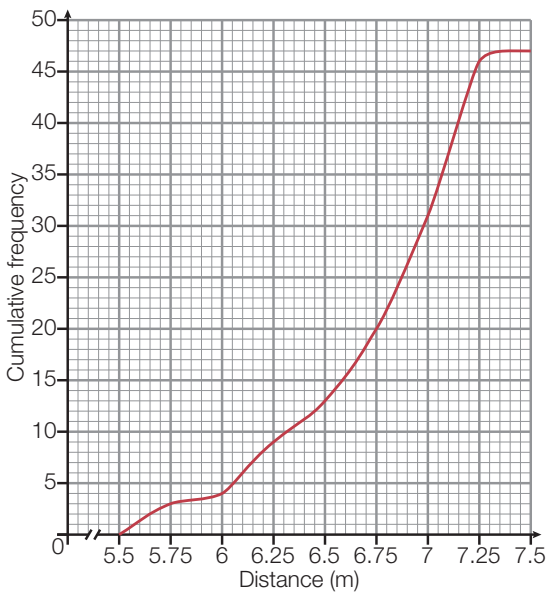
### Exercise 5.8

1. (a) 20, 50, 62, 76, 78, 78, 79, 81  
(b)



(c) 10.25 s

2. (a) 3, 4, 9, 13, 20, 31, 46, 47  
(b)



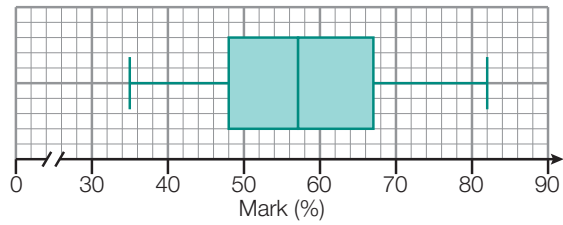
(c) 6.8 m

(d) 6.4 m, 7.1 m

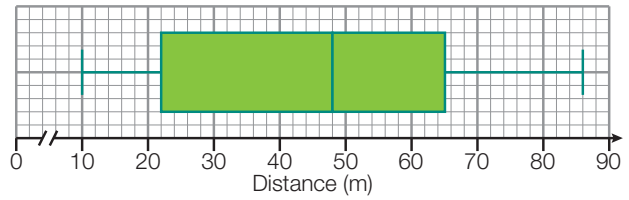
3. (a) 20.2 cm      (b) 43      (c) 11.7%

### Exercise 5.9

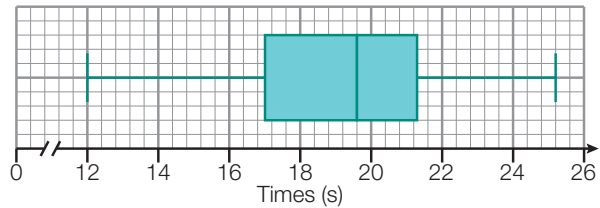
1. (a)



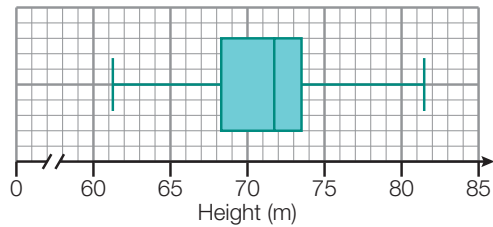
- (b)



- (c)

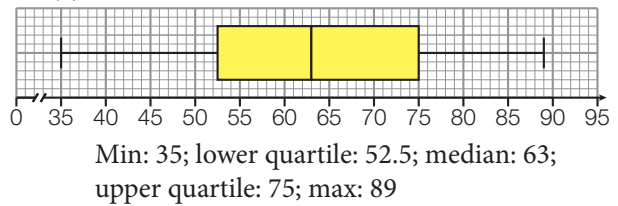


- (d)

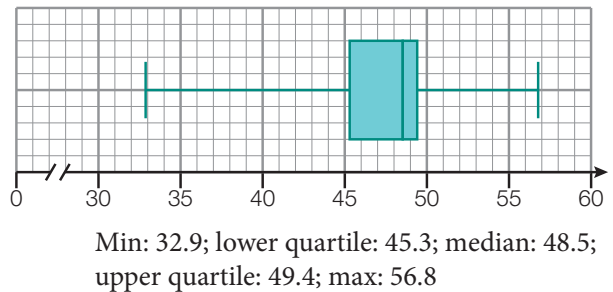


2. (Answers may vary, depending on estimates of the median and quartiles)

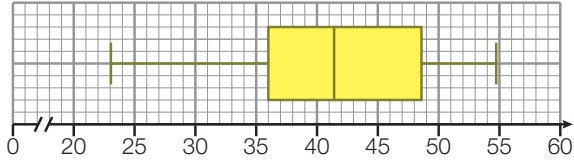
- (a)



- (b)

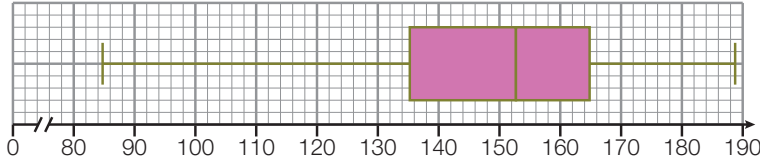


(c)



Min: 23.05; lower quartile: 36;  
median: 41.34; upper  
quartile: 48.66; max: 54.73

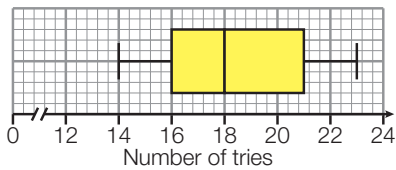
(d)



Min: 84.6; lower quartile: 135.6;  
median: 152.3; upper quartile:  
165.3; max: 188.4

### Exercise 5.10

- (a) Min 45, max 93  
(b) 74  
(c) 58  
(d) 87
- (a) 171 cm  
(b) Max 199 cm, min 154 cm  
(c)  $Q_1 = 160$  cm,  $Q_3 = 180$  cm
- (a) Frequencies: 6, 5, 5, 4, 7, 5,  
4, 6, 4, 4; total: 50  
(b)

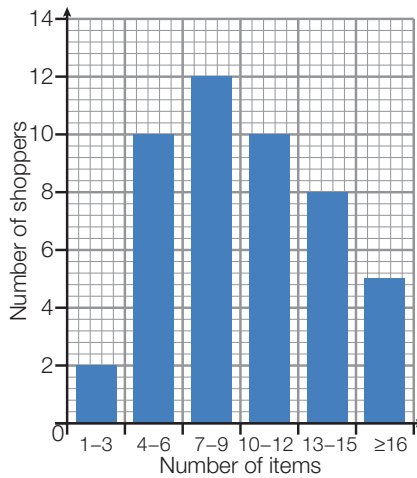


(c)  $Q_1 = 16$ , median = 18,  
 $Q_3 = 21$

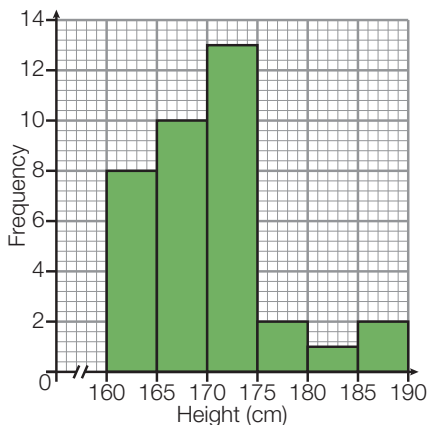
### Mixed examination practice 5

#### Exam-style questions 5

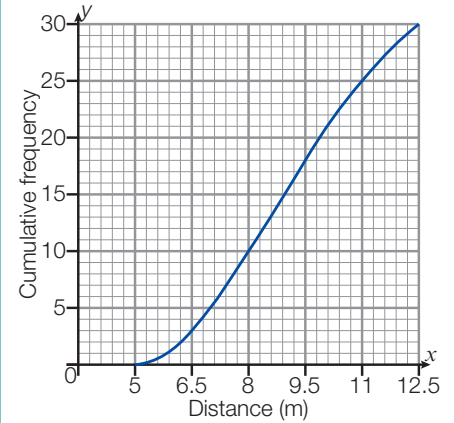
1.



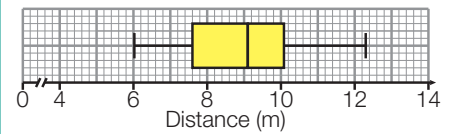
2. (a) Frequencies: 8, 10, 13, 2, 1, 2  
(b)



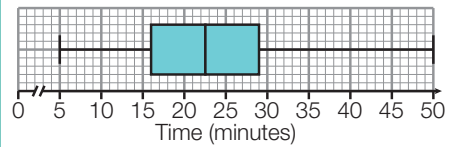
- (a) Frequencies: 3, 7, 8, 7, 5  
(b) Cumulative frequencies:  
3, 10, 18, 25, 30



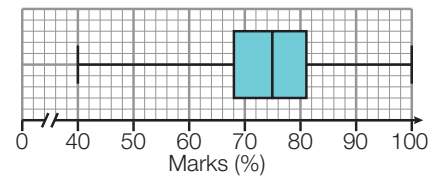
(c)



- (a) 22.5 minutes  
(b)  $Q_1 = 16$  minutes,  
 $Q_3 = 29$  minutes  
(c) 7  
(d)

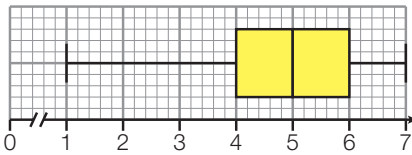


- (a) (i) 100  
(ii) 75%  
(iii)  $Q_1 = 68\%$ ,  $Q_3 = 81\%$   
(b) 41  
(c)



(d) 18

6. (a) 5  
 (b)  $Q_1 = 3, Q_3 = 5.5$   
 (c) 4  
 (d)



- (e) Median of A2 is higher than that of A1, indicating overall better marks. Marks of A2 are slightly more consistent than those of A1, as the difference  $Q_3 - Q_1$  is smaller.

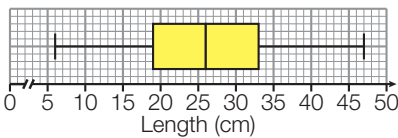
7. (a)

	Year 12G	Year 12H
Median	78	73
Lower quartile	60	65
Upper quartile	85	80

- (b) 12G had higher marks than 12H overall, but the test marks of 12H were more consistent, as shown by a smaller  $Q_3 - Q_1$  difference.

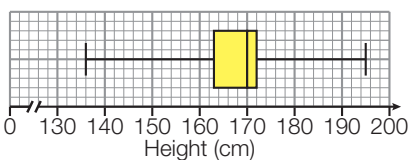
### Past paper questions 5

1. (a) 26 cm (b) 14 cm  
 (c)



2. (a) (ii) and (iv)  
 (b) (i) 10 (ii) 12  
 (iii) 0.8

3. (a) 170 cm  
 (b) 163 cm  
 (c) 172 cm  
 (d)



## Chapter 6

Answers for comparison or discussion questions are omitted as responses may vary.

### Exercise 6.1

1. 3  
 2. 6  
 3. £47

### Exercise 6.2

1. £170.52  
 2. \$79,640  
 3. 96.9 KB (3 s.f.)  
 4. (a) 4.14 years (3 s.f.)  
 (b) 118 lbs (3 s.f.)  
 5. (a) 307 (b) 12.8 (3 s.f.)  
 (c) 11.0 (3 s.f.)  
 6. (a)

	Type of car		
Average	4-Door Sedan	2-Door Coupe	Hatchback
Mean price(\$)	\$12,334.08	\$16,072.40	\$12,270
Median price (\$)	\$12,295	\$16,300	\$12,360

- (b) Mean price, because it makes use of all the data for each type of car  
 (c) Different sample sizes; use same/similar samples sizes
7. (a) 35 (b) 6.94 (3 s.f.)  
 (c) 7
8. (a) 51.5 million green tonnes  
 (b) 1287.5 thousand green tonnes
9. \$1.22
10. 10.41 seconds

### Exercise 6.3

1. (a) (i) 4 (ii) 3  
 (iii) 2  
 (b) (i) 64 (ii) 64  
 (iii) 57 and 70  
 (c) (i) 30.7 (3 s.f.)  
 (ii) 29  
 (iii) 29  
 (d) (i) 117.9 (ii) 110  
 (iii) 107 and 110  
 (e) (i) 42.1 (3 s.f.)  
 (ii) 42 (iii) 42 and 44
2. (a) 32 (b) 30.5  
 (c) 27
3. (a) 35 (b) 19  
 (c) 19
4. (a) (i) 5 (ii) 12.5  
 (iii) 10  
 (b) (i) 10.6 (ii) 19.6  
 (iii) 18 and 21.2  
 (c) (i) 5 (ii) 10.5  
 (iii) 7
5. (a) 25 (b) 47  
 (c) 1.88 (d) 0  
 (e) 1

6. (a)

	Round 1	Round 2	Rounds 1 and 2 combined
Mean	71.2	71.2	71.2
Mode	71 and 73	69	69 and 71
Median	71	71	71

7. (a) (i) 50.6 minutes (3 s.f.)  
(ii) 41–47 minutes  
(b) (i) 184 thousand  
(ii) 0–150 thousand

### Mixed examination practice 6

#### Exam-style questions 6

1. (a) 308  
(b) 54  
(c) 30
2. (a) Group 1: 27; Group 2: 25  
(b) Group 1: 4.81 (3 s.f.);  
Group 2: 4  
(d) 4.42 (3 s.f.)
3. 6.70 m (3 s.f.)
4. (a) 720 million  
(b) 15 million  
(c) 17.5 million (3 s.f.)
5. (a)  $x = 39, y = 48$   
(b)  $x = 13, y = 18$   
(c)  $x = 45, y = 46$
6. (a) 61 (b) 158  
(c) 2.59 (3 s.f.) (d) 2  
(e) 2
7. (a) 3.9 (b) 3 (c) 3

#### Past paper questions 6

1. (a) \$166 (to the nearest dollar)  
(b) \$165  
(c) (i) \$2430
2. (a) 10 (b) 4 (c) 6
3. (a) 51  
(b) (i) 60–70 cm  
(ii) 60–70 cm  
(iii) 69.5 cm (3 s.f.)

## Chapter 7

### Exercise 7.1

1. (a) (i) 6.8 (ii) 0.46  
(iii) 0.34  
(b) (i) 227 (ii) 27  
(iii) 12  
(c) (i) 72 (ii) 46  
(iii) 12  
(d) (i) 18.9 (ii) 19.2  
(iii) 7.8  
(e) (i) 330 (ii) 412  
(iii) 237  
(f) (i) 97.1 (ii) 3.5  
(iii) 1.4
2. (a) (i) 1 (ii) 5  
(iii) 2  
(b) (i) 165 cm  
(ii) 27 cm  
(iii) 18 cm  
(c) (i) 63 (ii) 6  
(iii) 2  
(d) (i) 3 (ii) 6  
(iii) 2  
(e) (i) 29 kg (ii) 7 kg  
(iii) 3 kg  
(f) (i) \$1849  
(ii) \$4450  
(iii) \$900
3. (a) (i) 47.5 years  
(ii) 39.5 years, 51.5 years  
(iii) 12 years  
(b) (i) 438 cm  
(ii) 432 cm, 445 cm  
(iii) 13 cm  
(c) (i) 95 (ii) 86, 102  
(iii) 16

### Exercise 7.2

1. (a)  $\bar{x} = 2.82$  hours (3 s.f.),  
 $\sigma_x = 2.08$  hours (3 s.f.)  
(b)  $\bar{x} = 0.239$  inches (3 s.f.),  
 $\sigma_x = 0.222$  inches (3 s.f.)  
(c)  $\bar{x} = 11.5^\circ\text{C}, \sigma_x = 0.782^\circ\text{C}$   
(d)  $\bar{x} = 19.2^\circ\text{C}, \sigma_x = 2.55^\circ\text{C}$
2. (a)  $18^\circ\text{C}$   
(b)  $1.69^\circ\text{C}$

3. (a) £2649.20  
(b) £711.91
4. (a)  $\bar{x} = 47\,100$  (3 s.f.),  
 $\sigma_x = 19\,300$  (3 s.f.)  
(b)  $\bar{x} = 39\,600$ ,  
 $\sigma_x = 21\,300$  (3 s.f.)
5.  $\bar{x} = 39, \sigma_x = 9.66$  (3 s.f.)

### Exercise 7.3

1. (a) (i) 2010: 649 (3 s.f.);  
2011: 662 (3 s.f.)  
(ii) 2010: 55.7 (3 s.f.);  
2011: 45.1 (3 s.f.)  
(b) On average more goals were scored per team in 2011, and the number of goals scored was less varied than in 2010.
2. (a) (i) MS-A: 50; MS-B: 53  
(ii) MS-A: quartiles 33 and 93, IQR = 60;  
MS-B: quartiles 50 and 70, IQR = 20  
(b) MS-B performed better, with both higher marks on average and more consistency.
3. (a), (b)

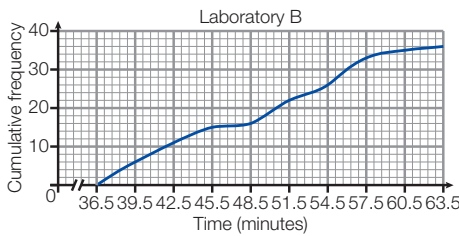
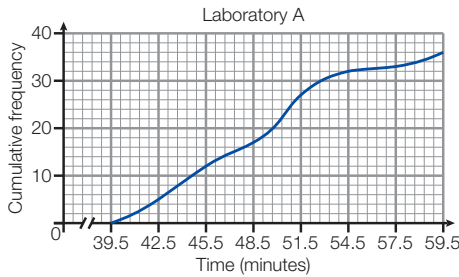
Lab A:

Time (minutes)	Frequency	Cumulative frequency
40–42	5	5
43–45	7	12
46–48	5	17
49–51	10	27
52–54	5	32
55–57	1	33
58–60	3	36

Lab B:

Time (minutes)	Frequency	Cumulative frequency
37–39	6	6
40–42	5	11
43–45	4	15
46–48	1	16
49–51	6	22
52–54	4	26
55–57	7	33
58–60	2	35
61–63	1	36

(c)



(d) Lab A: median 49.7, IQR 8;  
Lab B: median 49.8, IQR 14

(e) Although the two labs have approximately the same median waiting times, there is a much larger spread in waiting times at Lab B.

4. (a) 183 million  
(b) 193 million  
(c) 433 million (3 s.f.)

5. (a), (b), (c)

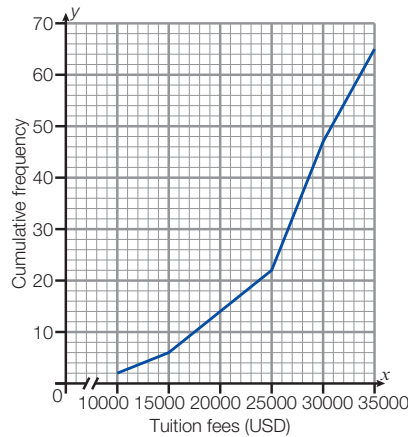
	M	F	M&F
$\bar{x}$	30.7	32.1	31.4
$\sigma$	20.4	21.2	20.8

(d) On average, females are older than males. The ages of males are slightly less varied than those of females.

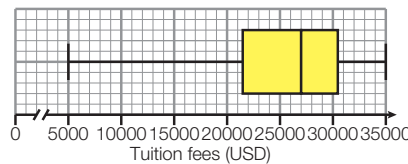
### Mixed examination practice 7

#### Exam-style questions 7

1. (a)  $x + y = 15$   
(b)  $x = 7, y = 8$   
(c) 8  
(d) 5.5 and 9  
(e) 3.5
2. (a) 25,000–30,000  
(b) 25,000–30,000  
(c) \$25,500  
(d)

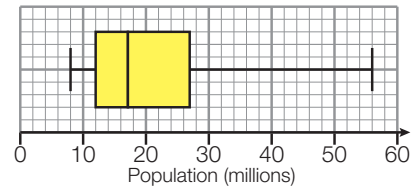


(e) (i) \$27,000 (ii) \$9000  
(f)

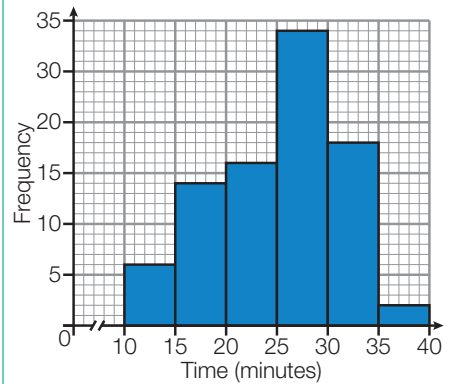


3.  $x = 22, y = 2$
4. (a) 50  
(b)  $c = 4, d = 8, e = 41, f = 3$
5. (a) 20.8 million barrels  
(b) 2.08 million barrels per year
6. (a) 31  
(b) 33.6

7. (a) (i) 17 million  
(ii) 46 (iii) 16%  
(b)

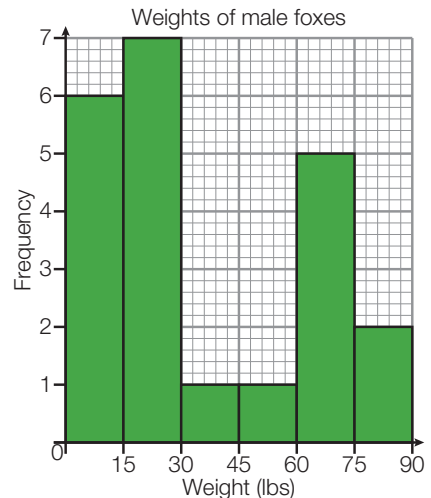


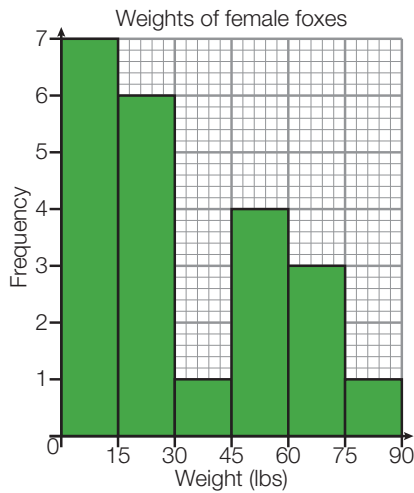
8. (a)  $25 \leq t < 30$   
(b)  $25 \leq t < 30$   
(c)  $\bar{x} = 25.3$  minutes (3 s.f.),  
 $\sigma_x = 6.11$  minutes (3 s.f.)  
(d)



(e) The mean lies near the lower boundary of the modal class, which is also the class in which the median lies. This reflects the fact that the data leans towards the lower values, as can also be seen from the histogram.

9. (a)

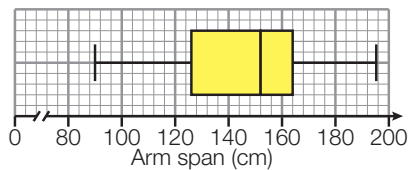




- (b) Males:  $15 \leq w < 30$ ;  
females:  $0 \leq w < 15$
- (c) Males:  $\bar{x} = 36.1$  lbs  
(3 s.f.),  $\sigma_x = 26.7$  lbs (3 s.f.);  
females:  $\bar{x} = 32.7$  lbs (3 s.f.),  
 $\sigma_x = 24.1$  lbs (3 s.f.)
- (d) The male and female  
median weights are similar,  
but the weights of the  
males are more varied than  
those of the females, with  
greater IQR.

10. (a) 60  
(b) (i) 130–150 km  
(ii) 130–150 km  
(c) (i) 146 km (3 s.f.)  
(ii) 30.4 km (3 s.f.)  
(d) All three averages lie  
within the same interval,  
and the standard deviation  
is only about 1.5 times the  
interval width, indicating  
a consistency among the  
data values.

11. (a) (i) 152 cm (ii) 38 cm  
(b)



- (c)  $u = 10$ ,  $v = 14$ ,  $w = 19$   
(d) 146 cm (3 s.f.)

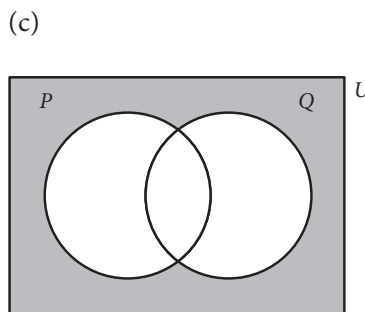
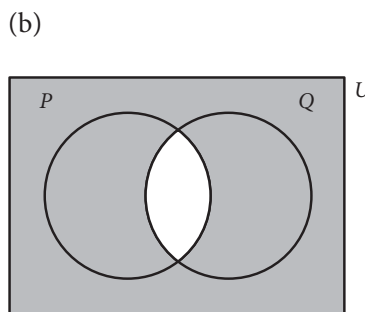
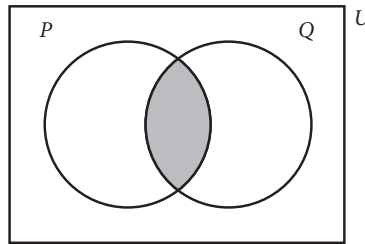
## Chapter 8

### Exercise 8.1

1. (a)  $x \in A$  (b)  $x \notin A$   
(c)  $B \subset C$  (d)  $C \cup D$   
(e)  $A \cap B$
2. (a)  $A = \{x, y, z\}$   
(b)  $n(A \cap B) = 3$   
(c)  $B = \{a, e, i, o, u\}$   
(d)  $n(A) = 5$

### Exercise 8.2

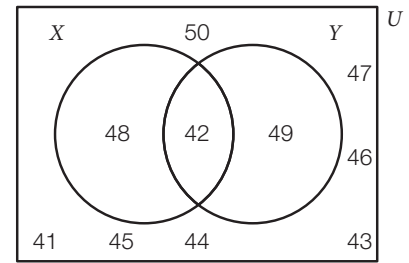
1. (a)  $n(P) = 8$   
(b)  $n(Q) = 6$   
(c)  $n(R) = 10$
2.  $A \cup B = \{5, 6, 7, \dots, 15\}$ ;  
 $A \cap B = \{9, 10, 11\}$
3. (a)



4. (a)  $A = \{1, 4, 9, 16, 25, 36, 49, 64\}$   
(b)  $B = \{1, 8, 27, 64\}$   
(c)  $A \cup B = \{1, 4, 8, 9, 16, 25, 27, 36, 49, 64\}$   
(d)  $A \cap B' = \{4, 9, 16, 25, 36, 49\}$

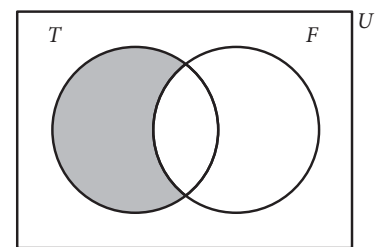
### Exercise 8.3

1. (a) (i) 42 (ii) 48  
(b) 9  
(c)

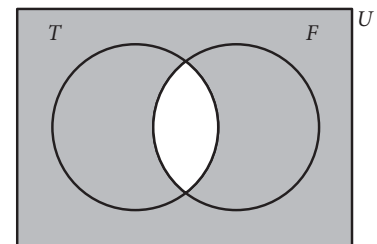


2. (a)  $A = \{5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53\}$   
(b) (i)  $B \cup C = \{5, 11, 22, 33, 44, 55\}$   
(ii)  $A \cap B \cup C = \{11\}$
3. (a) 8 (b)  $\{12, 36\}$   
(c)  $\{18\}$

4. (a) Track but not Field



- (b) Track or Field but not both



5. (a) (i) 6 (ii) 2  
(b) (i)  $\{11, 12\}$   
(ii)  $\{1, 8, 10\}$

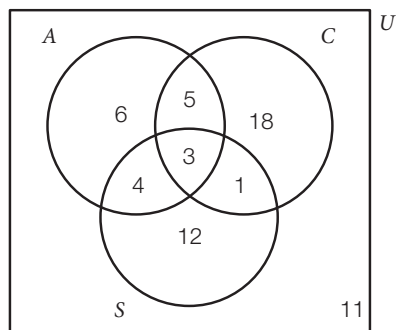


### Exercise 8.4

1. (a)  $P \cap Q \cap R$   
 (b)  $P \cap R$   
 (c)  $(P \cup R)' \cap Q$   
 (d)  $R \cap (P \cup Q)$

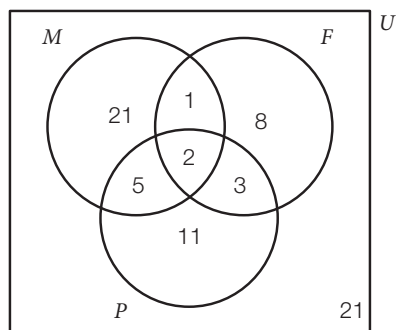
2. (a) {g}  
 (b) {a, e, f, h, j, k}  
 (c) {f, h, j}  
 (d) {c, f, g}

3. (a) and (b)



(c) 11

4. (a)

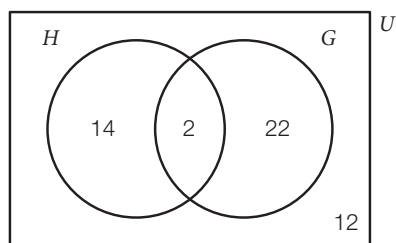


(b) 21

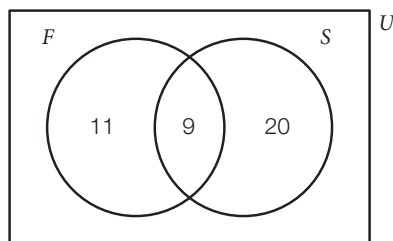
### Mixed examination practice 8

#### Exam-style questions 8

1. (a) Neither History nor Geography  
 (b) 48  
 (c)

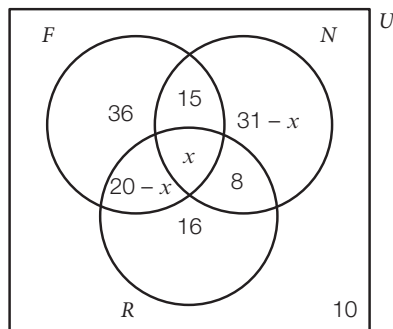


2. (a) 9  
 (b) Neither  $F$  nor  $S$ . Since  $n(F \cup S) = 40$ ,  $n(F \cup S)' = 0$   
 (c)



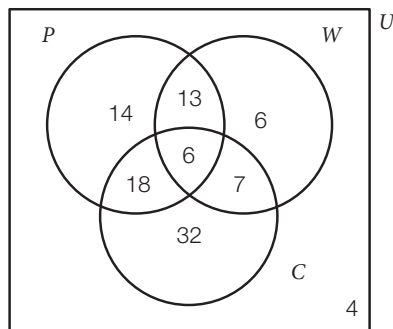
3. (a) (i) IB but not A Level  
 (ii)  $B \cap A'$   
 (b)  $x = 25$   
 (c) (i) 82 (ii) 12

4. (a) Given  $n(F \cap R) = 20$ .  
 Given that  $x$  read all 3. So, the region,  $(F \cap R \cap N') = 20 - x$   
 (b)  $31 - x$   
 (c)



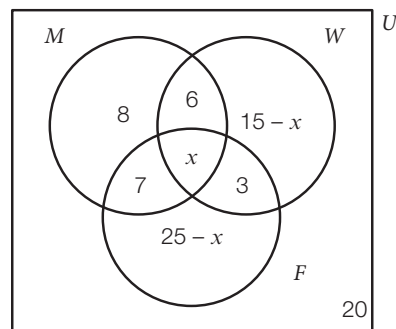
(d)  $x = 16$

- 5.

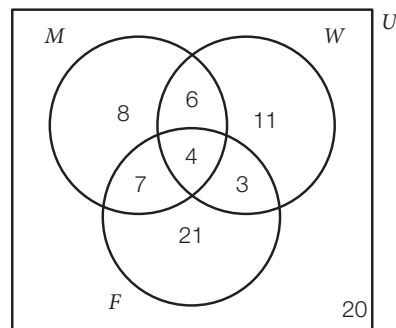


6. (a)  $x = 7$   
 (b) (i) 19 (ii) 144  
 (iii) 23 (iv) 98  
 7. (a) (i) 4 (ii) 40  
 (b) 42 (c) 70

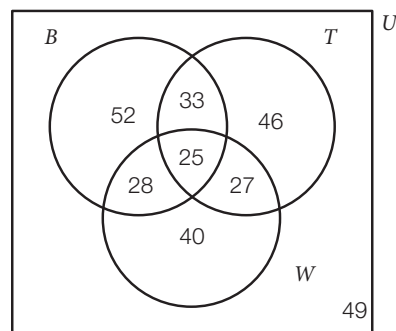
8. (a)



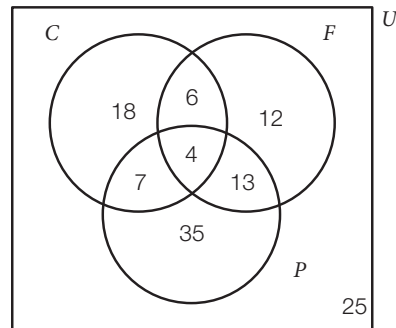
- (b)  $x = 4$  (c) 80  
 (d)



- 9.



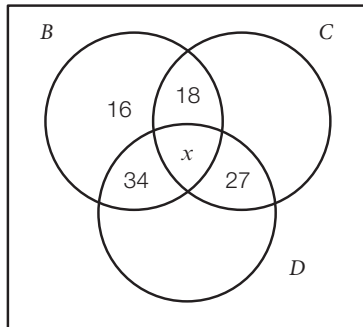
- 10.



#### Past paper questions 8

1. (a)  $20 - x$  (b)  $x = 15$   
 (c) 55

2. (a)



(b)  $x = 31$       (c) 156

## Chapter 9

### Exercise 9.1

- (a) Proposition, true  
(b) Not a proposition  
(c) Proposition, false  
(d) Proposition, true  
(e) Proposition, true  
(f) Not a proposition  
(g) Proposition, true  
(h) Not a proposition  
(i) Proposition, true  
(j) Not a proposition

### Exercise 9.2

- (a)  $\neg p$       (b)  $r \vee p$   
(c)  $q \wedge r$       (d)  $\neg r \wedge q$
- (a)  $x$  is not a prime number.  
(b)  $x$  is a prime number less than 100.  
(c) Either  $x$  is not a prime number or it is a 2-digit number.  
(d) Either  $x$  is not a prime number or it is not a 2-digit number.  
(e)  $x$  is not a prime number less than 100.
- (a) Jenny hates football and she does not watch Sky Sports.  
(b) Jenny does not hate football and she watches Sky Sports.  
(c) Either Jenny watches Sky Sports or she watches the Comedy Channel, but not both.

- (a)  $q \wedge p$       (b)  $\neg q \wedge \neg r$   
(c)  $\neg p \vee \neg q$       (d)  $\neg p \wedge \neg r$

### Exercise 9.3

- (a) Either Veejay attends football training or he passes his test, but not both.  
(b) Veejay revises for his test and he does not attend football training.  
(c) Veejay is not revising for his test and he attends football training.  
(d) If Veejay revises for his test then he does not attend football training.  
(e) If Veejay revises for his test then he passes his test.  
(f) If Veejay does not revise for his test then he does not pass his test.

- (a)  $p \Rightarrow r$       (b)  $q \Rightarrow \neg r$   
(c)  $\neg q \Rightarrow p$

- (a)  $p \Rightarrow q$       (b)  $\neg q \Rightarrow \neg p$   
(c)  $p \Leftrightarrow q$

- (a) If  $x$  is a quadrilateral and also a 2-D shape with a pair of parallel sides then  $x$  is a parallelogram. False.  
(b) If  $x$  is a parallelogram then  $x$  is a quadrilateral. True.  
(c) If  $x$  is a parallelogram then  $x$  is a 2-D shape with a pair of parallel sides. True.  
(d)  $x$  is a quadrilateral and also a 2-D shape with a pair of parallel sides if and only if  $x$  is a parallelogram. False.

### Exercise 9.4

- (a) (i)  $p \wedge q$       (ii)  $\neg p \wedge \neg q$   
(b) Either Donald did not pass his driving test or Debbie passed her driving test.

(c)

$p$	$q$	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

- (a) (i)  $\neg p \Rightarrow \neg q$   
(ii)  $q \Leftrightarrow p$   
(b)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

### Exercise 9.5

- (a)  $m \wedge (e \vee s)$   
(b) If a student does not choose Science then he/she chooses Economics.

(c)

$e$	$s$	$\neg e$	$\neg e \Rightarrow s$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

- (a) If I go to the cinema then it is not the weekend.

(b)

$p$	$q$	$\neg p$	$q \Rightarrow \neg p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

3.

$p$	$q$	$r$	$p \vee q$	$r \Rightarrow (p \vee q)$
T	T	T	F	F
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	F	T

4. (a) (i)  $p \Rightarrow q$  (ii)  $q \wedge \neg r$   
 (iii)  $\neg p \Rightarrow r$   
 (b) If Boris does not have a football then either he is a rugby player or he has the rugby ball.

(c)

$p$	$q$	$r$	$\neg r$	$p \vee q$	$\neg r \Rightarrow (p \vee q)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	F

### Exercise 9.6

1. (a) (i)

$p$	$q$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

(ii) Neither

(b) (i)

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

(ii) Tautology

(c) (i)

$p$	$q$	$\neg p$	$p \Rightarrow q$	$\neg p \wedge q$	$(p \Rightarrow q) \wedge (\neg p \wedge q)$
T	T	F	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	F

(ii) Neither

(d) (i)

$p$	$q$	$\neg p$	$p \Rightarrow q$	$\neg(p \Rightarrow q)$	$\neg p \vee q$	$\neg(p \Rightarrow q) \Leftrightarrow \neg p \vee q$
T	T	F	T	F	T	F
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	F	T	T	F	T	F

(ii) Contradiction

(e) (i)

$p$	$q$	$\neg q$	$\neg q \wedge p$	$p \wedge (\neg q \wedge p)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	F

(ii) Neither

2. (a) (i) Either the bad weather does not continue or this week's cricket match will be cancelled.  
 (ii) The bad weather does not continue and this week's cricket match is not cancelled.  
 (iii) This week's cricket match will not be cancelled if and only if the bad weather does not continue.  
 (iv) Either the bad weather continues and this week's cricket match is cancelled or the cricket match will be cancelled and the bad weather does not continue (not both).

(b) (i) Logically equivalent

$p$	$q$	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p$	$q$	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(ii) Logically equivalent

$p$	$q$	$r$	$(p \wedge q)$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$p$	$q$	$r$	$(q \wedge r)$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(iii) Logically equivalent

$p$	$q$	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg q \wedge \neg p$	$(p \wedge q) \vee (\neg q \wedge \neg p)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

(iv) Not logically equivalent

$p$	$q$	$r$	$p \Rightarrow q$	$(p \Rightarrow q) \Rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

$p$	$q$	$r$	$q \Rightarrow r$	$q \Rightarrow (q \Rightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

3. (a) (i) If the internet is not working then I do not check my emails.

(ii) If the internet is working then I check my emails.

(b)

$p$	$q$	$\neg p$	$\neg q$	$p \Rightarrow \neg q$	$\neg p \Rightarrow q$	$(p \Rightarrow \neg q) \vee (\neg p \Rightarrow q)$
T	T	F	F	F	T	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	F	T

(c) Statement is a tautology.

## Exercise 9.7

- $\neg p \Rightarrow \neg r$  means: if the music is not good then I do not dance to it.
  - $r \Rightarrow p$  means: if I dance to the music then the music is good.
  - $\neg r \Rightarrow \neg q$  means: if I do not dance to the music then I do not feel like dancing.
- If you do not listen attentively in class, then you do not perform well in tests.
    - If you perform well in tests then you listen attentively in class.
    - If you do not perform well in tests then you do not listen attentively in class.
  - If you do not like current affairs, then you do not listen to news regularly.
    - If you listen to news regularly then you like current affairs.
    - If you do not listen to news regularly then you do not like current affairs.
  - If you are not taught by Mrs Brown, then you are not brilliant at Logic.
    - If you are brilliant at Logic then you are taught by Mrs Brown.
    - If you are not brilliant at Logic then you are not taught by Mrs Brown.
  - If Sandra is not unwell, then she can play in the netball match.
    - If Sandra cannot play in the netball match then she is unwell.

- If Sandra can play in the netball match then she is not unwell.
  - If Andrew is not good at languages, then he cannot be a tourist guide.
  - If Andrew can be a tourist guide then he is good at languages.
  - If Andrew cannot be a tourist guide then he is not good at languages.

- If Grandma visits Aunt Sally then she goes to the dentist.
    - $\neg q \Rightarrow p$
    - No; it is none of these.
  - If a shape is a parallelogram then it is a rectangle.
    - If a shape is not a rectangle then it is not a parallelogram.
    - If a shape is not a parallelogram then it is not a rectangle.
    - Statement (iii) is true.

## Mixed examination practice 9

### Exam-style questions 9

- My laptop is not fixed and I will not finish writing up my Portfolio task.
    - If my laptop is fixed then I will finish writing up my portfolio task.
    - I will finish writing up my portfolio task if and only if my laptop is fixed.
  - $p \wedge \neg q$
    - $p \wedge \neg r$
- If New Year is approaching then I will shop for presents.

(b)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

- If I save enough money then I buy a new car.
  - $\neg p \wedge \neg q$
  -

$p$	$q$	$\neg p$	$\neg q$	$\neg p \Rightarrow q$	$\neg p \wedge \neg q$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	F	T

- If Elliot does not pass his driving test then his dad will not buy him a new car.
    - If his dad buys him a new car then Elliot passes his driving test.
    - If his dad does not buy him a new car then Elliot does not pass his driving test.
  - If it does not snow heavily tonight then the roads will be busy tomorrow morning.
    - If the roads are not busy tomorrow morning then it will snow heavily tonight.
    - If the roads are busy tomorrow morning then it will not snow heavily tonight.
  - If the recession does not continue then unemployment will not remain high.
    - If unemployment remains high then the recession will continue.

(iii) If unemployment does not remain high then the recession will not continue.

5. (a)  $p \Rightarrow \neg q$   
 (b)  $q \Rightarrow \neg p$

6. (a) If  $x$  has equal sides and equal angles then  $x$  is a regular polygon. False.  
 (b)  $x$  is a regular polygon if and only if  $x$  is a polygon with equal sides and equal angles. True.  
 (c)  $x$  is a polygon if and only if  $x$  has equal sides and equal angles. False.

7. (a) (i) If Marco is not a member of the debating society then he does not enjoy debating.

(ii) If Marco enjoys debating then he is a member of the debating society.

(b) (i)

$p$	$q$	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(ii)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(iii)

$p$	$q$	$\neg q$	$p \vee \neg q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

(iv)

$p$	$q$	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

- (c) Statements (ii) and (iii) are logically equivalent.

### Past paper questions 9

1.

$p$	$q$	$\neg q$	$p \wedge \neg q$	$p \vee q$	$(p \wedge \neg q) \Rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

2. (a)

$p$	$q$	$p \wedge q$	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$\Rightarrow$	$q$
T	T	T	T	F	F	T	T
T	F	F	T	F	F	T	F
F	T	F	T	T	T	T	T
F	F	F	F	T	F	T	F

(b) Valid argument or tautology

3. (a) Both are ' $p$  or  $q$ '; the first is 'but not both'.

(b)

$\neg q$	$p \vee q$	$\neg p \vee \neg q$	$p \vee q \Rightarrow \neg p \vee \neg q$
F	F	F	T
T	T	T	T
F	T	T	T
T	F	F	T

(c) Tautology

$p$	$q$	$\neg p$
T	T	F
T	F	F
F	T	T
F	F	T

4. (a) (i)

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

(ii) Yes

(b)  $p \vee q$

## Chapter 10

### Exercise 10.1

- 0.01
- (a)  $\frac{5}{20}$  (b)  $\frac{15}{20}$
- (a)  $\frac{3}{12}$  (b)  $\frac{9}{12}$

### Exercise 10.2

1.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

2.

Head	Dep. H
B	P
B	R
P	B
P	R
R	B
R	P

3.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

4. (a) 9  
(b)

1	A	A	B	B	B	C	C	C
2	B	C	B	A	C	C	A	B

(c) 27

### Exercise 10.3A

1. (a)  $\frac{8}{20}$  (b)  $\frac{12}{20}$  (c)  $\frac{13}{20}$

2.

	1	2	3	4	5	6
Head	H1	H2	H3	H4	H5	H6
Tail	T1	T2	T3	T4	T5	T6

(a)  $\frac{6}{12}$  (b)  $\frac{8}{12}$

(c)  $\frac{2}{12}$  (d)  $\frac{9}{12}$

3. (a)  $\frac{2}{11}$  (b)  $\frac{2}{11}$  (c)  $\frac{7}{11}$

4. (a)  $\frac{2}{36}$  (b)  $\frac{33}{36}$

(c)  $\frac{7}{36}$  (d)  $\frac{15}{36}$

5. (a) WW, WL, WD, LW, LL,  
LD, DW, DL, DD

(b)  $\frac{4}{9}$

(c)  $\frac{4}{9}$

6. (a)  $\frac{8}{30}$  (b)  $\frac{17}{30}$  (c)  $\frac{5}{30}$

7.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(a)  $\frac{8}{36}$  (b)  $\frac{6}{36}$  (c)  $\frac{3}{36}$

(d)  $\frac{20}{36}$  (e)  $\frac{4}{36}$

8. (a) 80

(b) (i)  $\frac{50}{80}$  (ii)  $\frac{15}{80}$  (iii)  $\frac{55}{80}$

9. (a)

	Andrew	Fareeda	Caitlin
	HL	HL	HL
	HL	HL	SL
	HL	SL	SL
	HL	SL	HL
	SL	SL	SL
	SL	SL	HL
	SL	HL	HL
	SL	HL	SL

(b) 8 (c)  $\frac{2}{8}$

(d)  $\frac{3}{8}$  (e)  $\frac{7}{8}$

### Exercise 10.3B

1. HH, HT, TH, TT  
(a)  $\frac{1}{4}$  (b)  $\frac{3}{4}$  (c) 60  
(d) 30 (e) 90

2. HHH, HHT, HTH, HTT,  
TTH, THT, TTH, TTT  
(a)  $\frac{1}{8}$  (b)  $\frac{3}{8}$  (c)  $\frac{7}{8}$   
(d)  $\frac{7}{8}$  (e) 12 (f) 84

3. (a) (i)  $\frac{120}{300}$  (ii)  $\frac{165}{300}$   
(b) (i) 52 (ii) 143

4. (a) (i)  $\frac{24}{100}$  (ii)  $\frac{74}{100}$   
(b) (i) 209 (ii) 814

### Exercise 10.4

1. (a)  $\frac{12}{32}$  (b)  $\frac{20}{32}$

2. (a)  $\frac{6}{16}$  (b)  $\frac{3}{16}$  (c)  $\frac{9}{16}$

3. (a)  $\frac{6}{24}$  (b)  $\frac{4}{24}$   
(c)  $\frac{10}{24}$  (d)  $\frac{18}{24}$

4. (a) 0.15 (b) 0.85 (c) 0.75

5. (a)  $\frac{5}{28}$  (b)  $\frac{19}{28}$  (c)  $\frac{9}{28}$

6. (a) 1.2  
(b) (i) 0.698  
(ii) 0.151  
(iii) 0.232  
(c) (i) 464 (ii) 1396

### Exercise 10.5

1. (a) 0.54  
(b) Not independent;  
 $P(A) \times P(B) = 0.2706 \neq 0.54$

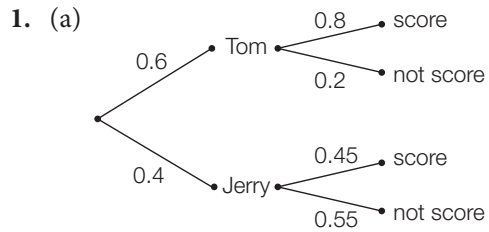
2. 0.83

3. (a) 0.2975  
(b) 0.9475

4. (a) (i)  $\frac{174}{260}$  (0.669)  
(ii)  $\frac{5}{260}$  (0.0192)  
(b) (i) 0.379  
(ii) 0.810

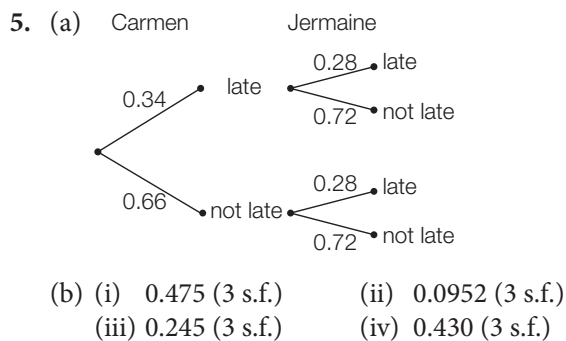
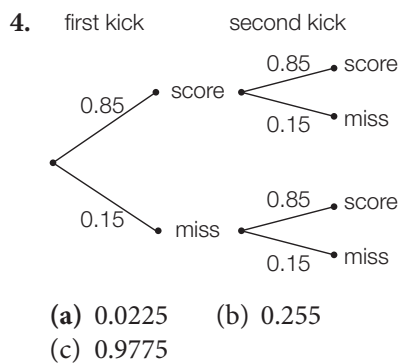
5. (a) 0.0125 (3 s.f.) (b) 0.150 (3 s.f.)  
 (c) 0.881 (3 s.f.)
6. (a)  $\frac{3}{12}$  (b)  $\frac{5}{12}$
7. (a) 0.655 (3 s.f.) (b) 0.345 (3 s.f.)  
 (c) 0.891 (3 s.f.) (d) 0.818 (3 s.f.)

### Exercise 10.6A

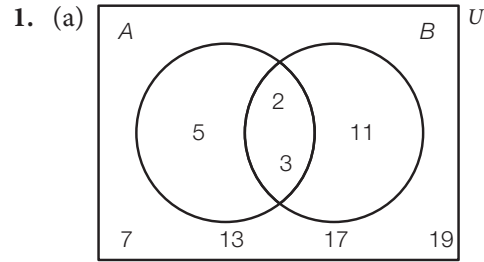


- (b) (i) 0.48 (ii) 0.66  
 (iii) 0.34
- (c) (i) 0.495 (ii) 0.7975
2. (a) 0.33 (b) 0.18 (c) 0.82
3. (a)
- 
- ```

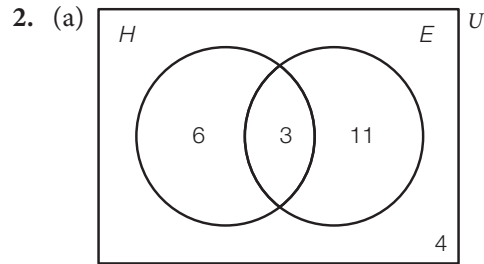
  graph LR
    Root(( )) ---|0.3| Midfielder
    Root ---|0.7| Striker
    Midfielder ---|0.6| MidfielderScore[score]
    Midfielder ---|0.4| MidfielderNotScore[not score]
    Striker ---|0.9| StrikerScore[score]
    Striker ---|0.1| StrikerNotScore[not score]
  
```
- (b) 0.18 (c) 0.81  
 (d) 0.07 (e) 0.19



### Exercise 10.6B



- (b) (i)  $\frac{2}{8}$  (ii)  $\frac{2}{8}$



- (b) (i)  $\frac{20}{24}$  (ii)  $\frac{10}{24}$  (iii)  $\frac{17}{24}$

3. (a) 80  
 (b) (i)  $\frac{37}{80}$  (ii)  $\frac{23}{80}$  (iii)  $\frac{39}{80}$   
 (iv)  $\frac{50}{80}$  (v)  $\frac{30}{80}$

### Exercise 10.7

1. (a)  $\frac{12}{13}$  (b)  $\frac{6}{13}$  (c)  $\frac{6}{13}$
2. (a)  $\frac{4}{25}$  (b)  $\frac{12}{25}$
3. (a)  $\frac{28}{153}$  (0.183) (b)  $\frac{45}{153}$  (0.294)  
 (c)  $\frac{125}{153}$  (0.817)
4. (a)  $\frac{83}{225}$  (0.369) (b)  $\frac{142}{225}$  (0.631)  
 (c)  $\frac{81}{225}$  (0.36)
5. (a)  $\frac{14}{95}$  (0.147) (b)  $\frac{81}{95}$  (0.853)  
 (c)  $\frac{48}{95}$  (0.505)
6. (a)  $\frac{10}{32}$  (0.3125) (b)  $\frac{30}{32}$  (0.9375)  
 (c)  $\frac{3}{496}$  (0.00605) (d)  $\frac{87}{496}$  (0.175)  
 (e)  $\frac{90}{496}$  (0.181)



7. (a)

|    | R | Y | G  | Br | Bl | P  | Bk |
|----|---|---|----|----|----|----|----|
| R  | - | 3 | 4  | 5  | 6  | 7  | 8  |
| Y  | 3 | - | 5  | 6  | 7  | 8  | 9  |
| G  | 4 | 5 | -  | 7  | 8  | 9  | 10 |
| Br | 5 | 6 | 7  | -  | 9  | 10 | 11 |
| Bl | 6 | 7 | 8  | 9  | -  | 11 | 12 |
| P  | 7 | 8 | 9  | 10 | 11 | -  | 13 |
| Bk | 8 | 9 | 10 | 11 | 12 | 13 | -  |

(b) (i)  $\frac{6}{42}$  (ii)  $\frac{34}{42}$  (iii)  $\frac{4}{42}$

8. (a) (i)  $\frac{110}{140}$  (ii)  $\frac{77}{140}$  (iii)  $\frac{46}{140}$

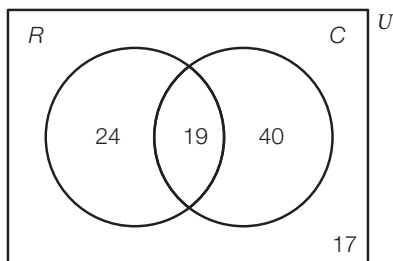
(b) (i) 0.0577 (3 s.f.)  
 (ii) 0.412 (3 s.f.)  
 (iii) 0.384 (3 s.f.)

### Exercise 10.8

1. (a) (i)  $\frac{26}{100}$  (ii)  $\frac{12}{40}$

(b)  $\frac{46}{60}$

2. (a)



(b) 19

(c) (i)  $\frac{19}{100}$  (ii)  $\frac{40}{57}$  (iii)  $\frac{19}{59}$

3. (a)  $\frac{42}{150}$  (b)  $\frac{42}{67}$  (c)  $\frac{18}{43}$

(b) (i)  $\frac{66}{140}$  (ii)  $\frac{68}{74}$  (iii)  $\frac{24}{30}$

8. (a) 82

(b) (i) 0.179 (3 s.f.)  
 (ii) 0.257 (3 s.f.)  
 (iii) 0.459 (3 s.f.)

9. (a)

|            | M  | F  | Total |
|------------|----|----|-------|
| Baseball   | 29 | 10 | 39    |
| Basketball | 20 | 18 | 38    |
| Hockey     | 14 | 19 | 33    |
| Total      | 63 | 47 | 110   |

(b) (i) 0.424 (3 s.f.)  
 (ii) 0.0416 (3 s.f.)  
 (iii) 0.226 (3 s.f.)

10. (a) 0.702 (3 s.f.)

(b) 0.538 (3 s.f.)  
 (c) (i) 0.0423 (3 s.f.)  
 (ii) 0.605 (3 s.f.)

11. (a) (i) 0.1

(ii) 0.4  
 (b) (i) 0.648 (3 s.f.)  
 (ii) 0.133 (3 s.f.)

### Mixed examination practice 10

#### Exam-style questions 10

1. (a) 0.1 (b) 0.2 (c) 0.8

2.  $\frac{4}{8}$

3. (a)  $\frac{2}{9}$  (b)  $\frac{1}{9}$  (c)  $\frac{3}{9}$

4. (a)  $\frac{17}{60}$  (b)  $\frac{23}{60}$  (c)  $\frac{40}{60}$

5. (a)

|                  | Over 18's | 18 or under | Total |
|------------------|-----------|-------------|-------|
| Have cars        | 20        | 12          | 32    |
| Do not have cars | 28        | 20          | 48    |
| Total            | 48        | 32          | 80    |

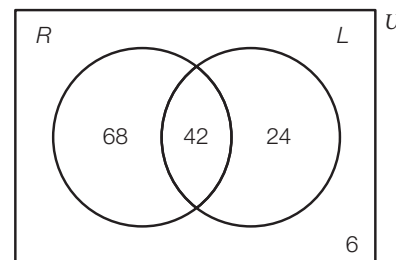
(b)  $\frac{12}{80}$  (c)  $\frac{48}{80}$

6. GG, GS, GB, SG, SS, SB, BG, BS, BB

(a) 9

(b) (i)  $\frac{1}{9}$  (ii)  $\frac{4}{9}$  (iii)  $\frac{6}{9}$

7. (a)



12. (a) (i) Walk, given that they rode to school by bike.

(ii) Travelled by car, given that they did not ride by bike.

(iii) Did not ride by bike, given that they walked and travelled by car.

(b) (i)  $\frac{5}{13}$

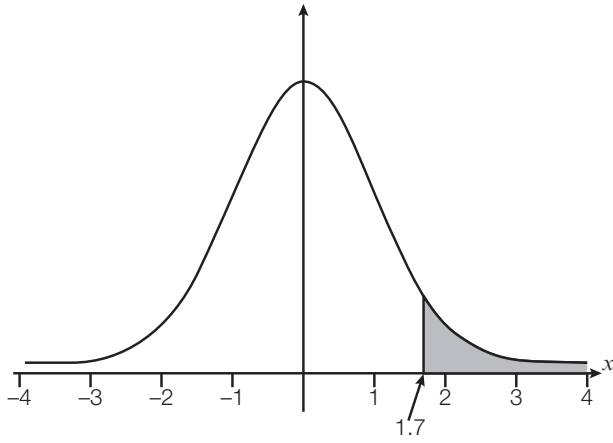
(ii)  $\frac{27}{35}$

(iii)  $\frac{4}{5}$

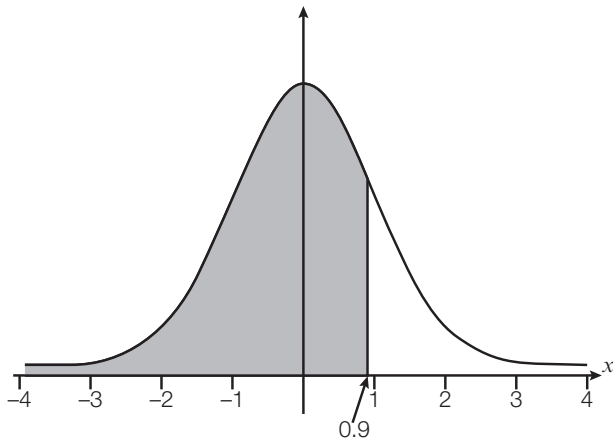
# Chapter 11

## Exercise 11.1

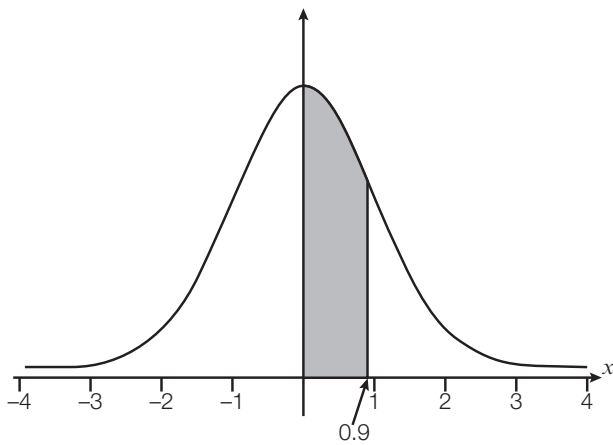
1. (a)



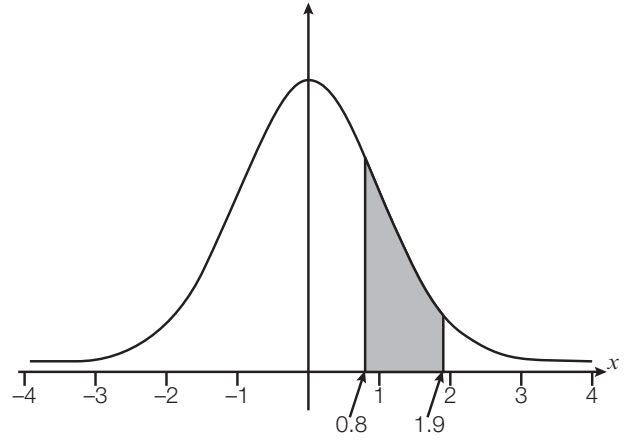
(b)



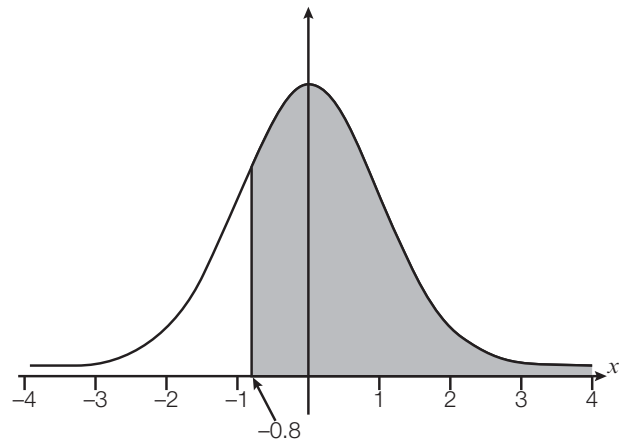
(c)



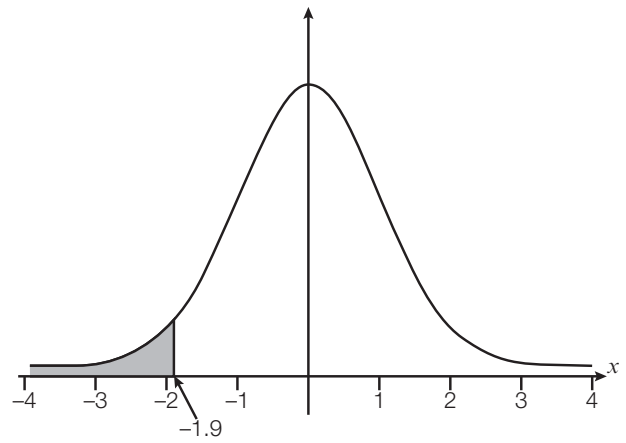
(d)



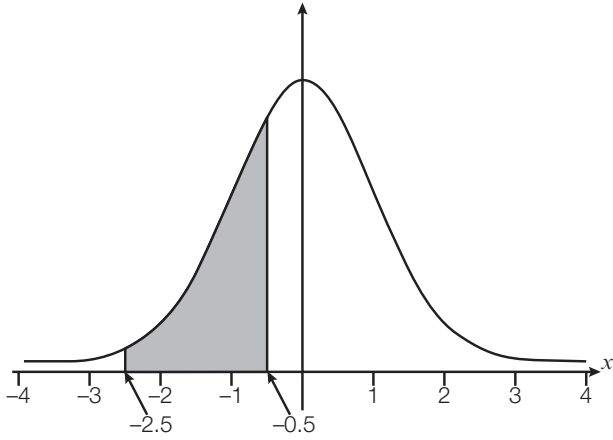
(e)



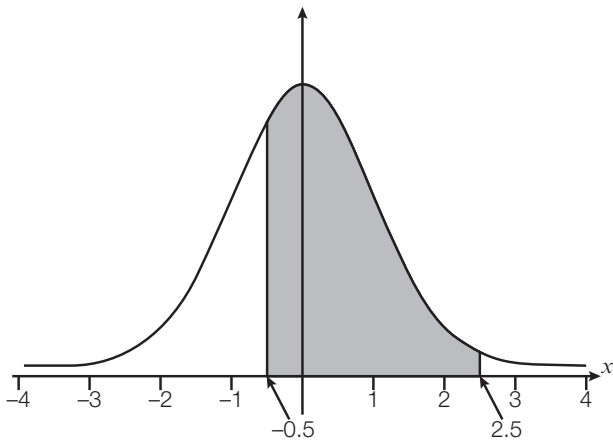
(f)



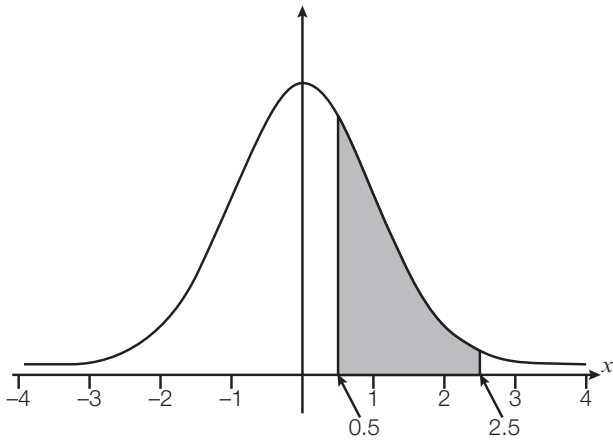
(g)



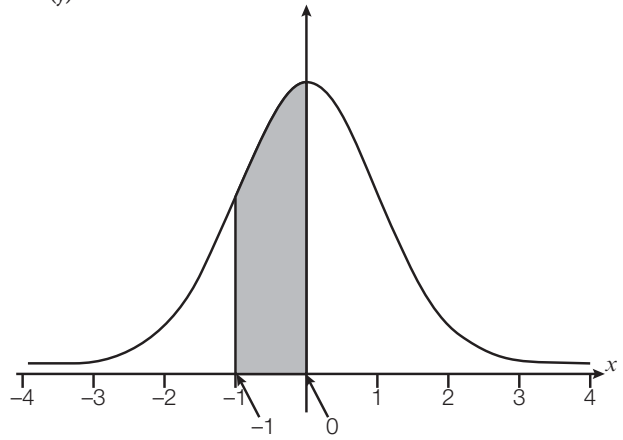
(h)



(i)

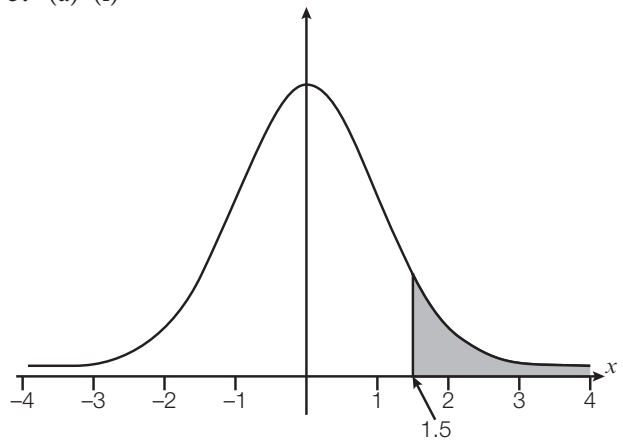


(j)

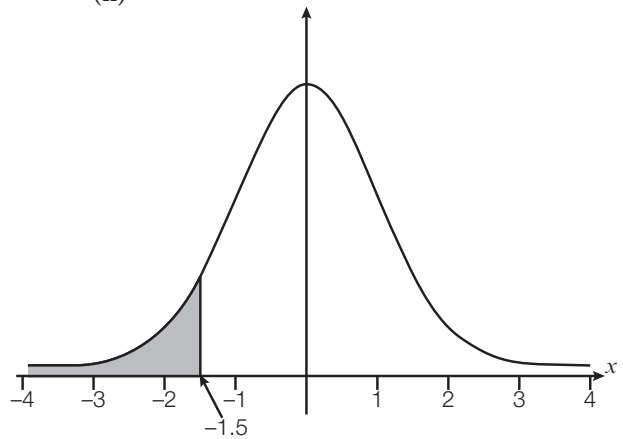


2. (a) 0.0446 (3 s.f.)      (b) 0.816 (3 s.f.)  
 (c) 0.316 (3 s.f.)      (d) 0.183 (3 s.f.)  
 (e) 0.788 (3 s.f.)      (f) 0.0287 (3 s.f.)  
 (g) 0.302 (3 s.f.)      (h) 0.685 (3 s.f.)  
 (i) 0.302 (3 s.f.)      (j) 0.341 (3 s.f.)

3. (a) (i)



(ii)

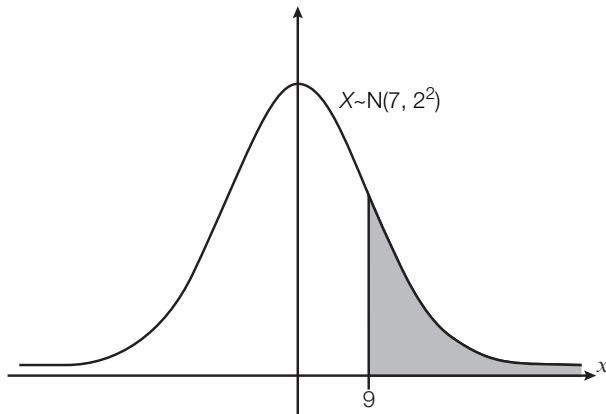


- (b) Areas are equal.      (c) 0.0668 (3 s.f.)

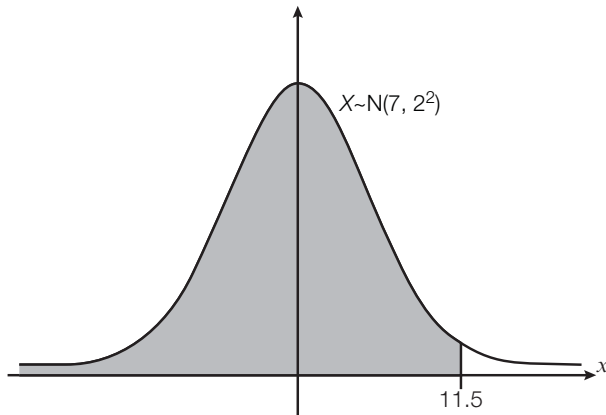
4. Their sum is equal to 1.

## Exercise 11.2

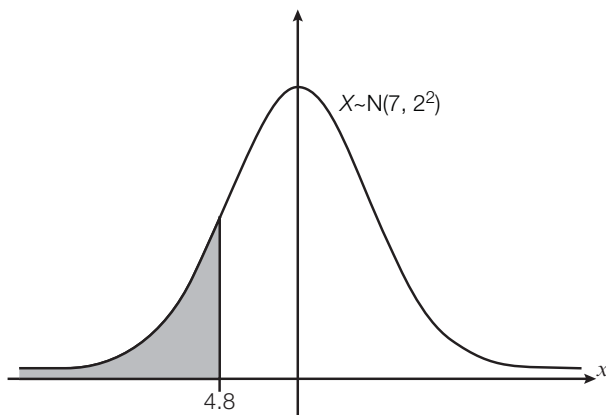
1. (a) (i)



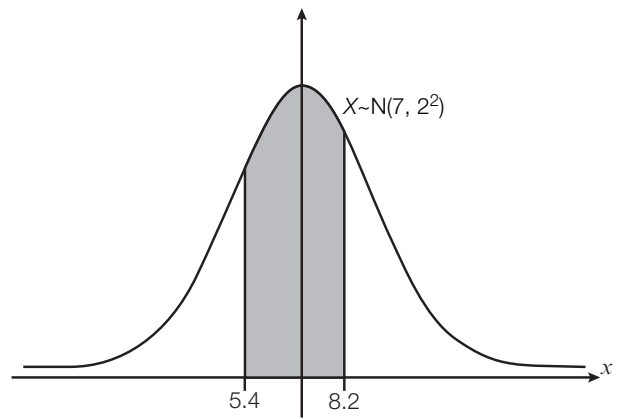
(ii)



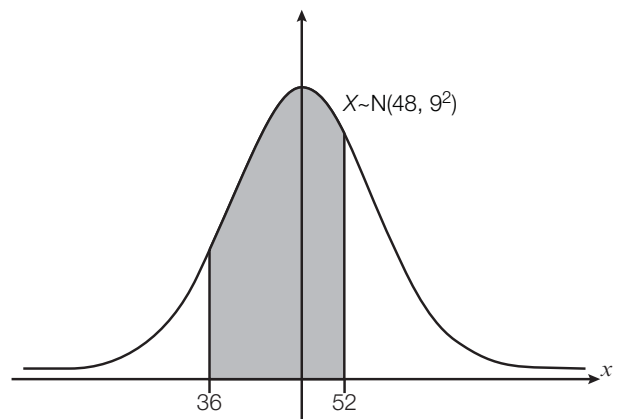
(iii)



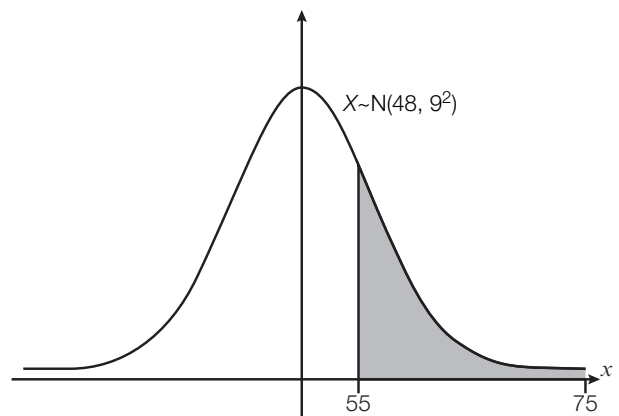
(iv)



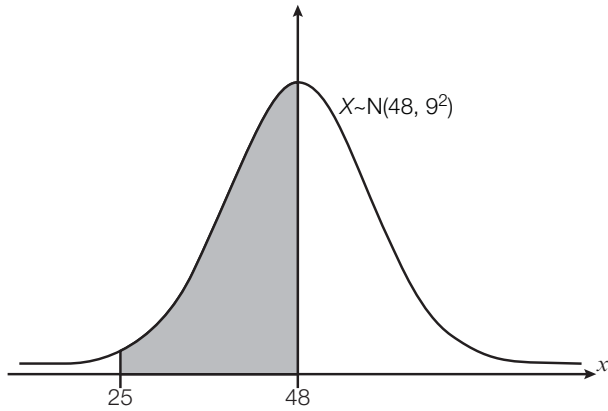
(b) (i)



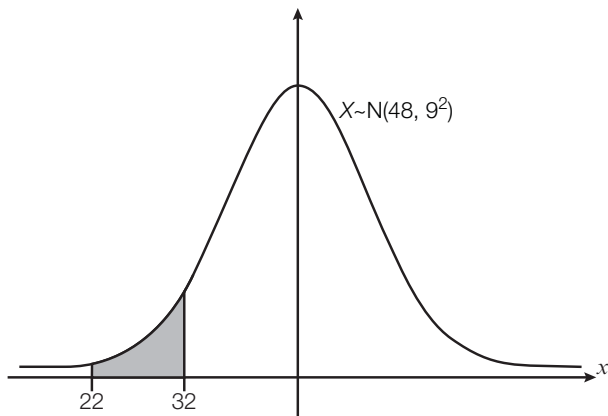
(ii)



(iii)



(iv)



2. (a) (i) 0.159 (3 s.f.) (ii) 0.988 (3 s.f.)  
 (iii) 0.136 (3 s.f.) (iv) 0.514 (3 s.f.)  
 (b) (i) 0.580 (3 s.f.) (ii) 0.217 (3 s.f.)  
 (iii) 0.495 (3 s.f.) (iv) 0.0358 (3 s.f.)
3. (a) 0.0712 (3 s.f.) (b) 0.141 (3 s.f.)  
 (c) 0.429 (3 s.f.) (d) 0.791 (3 s.f.)  
 (e) 0.136 (3 s.f.)
4. 0.0668 (3 s.f.)

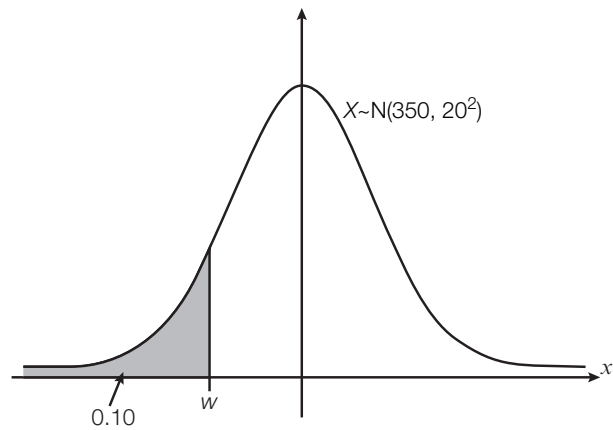
### Exercise 11.3

1. (a) 0.922 (3 s.f.)  
 (b) 0.653 (3 s.f.)  
 (c) 0.972 (3 s.f.)
2. (a) (i) 0.894 (3 s.f.)  
 (ii) 0.994 (3 s.f.)  
 (iii) 0.733 (3 s.f.)  
 (b) (i) 96 (ii) all 100  
 (c) No
3. (a) 0.330 (3 s.f.) (b) 67.6% (3 s.f.)  
 (c) 0.153 (3 s.f.) (d) 92

4. (a) 97.7% (3 s.f.) (b) 97.7% (3 s.f.)  
 (c) 11.3% (3 s.f.)
5. (a) 16 (b) 3 (c) 126 (d) 5
6. (a) (i) 0.952 (3 s.f.) (ii) 0.00621 (3 s.f.)  
 (iii) 0.894 (3 s.f.) (iv) 0.0478 (3 s.f.)  
 (b) (i) 1401 (ii) 67

### Exercise 11.4

1. 0.961 (3 s.f.)  
 2. 0.841 (3 s.f.)  
 3. (a) 0.141 (3 s.f.) (b) 72.8 (3 s.f.)  
 4. 0.775 (3 s.f.)  
 5. (a) (i) 0.0668 (3 s.f.) (ii) 0.977 (3 s.f.)  
 (iii) 0.930 (3 s.f.)  
 (b) (i)



- (ii) 324 (3 s.f.)
6.  $x = 141$  (3 s.f.),  $y = 179$  (3 s.f.)
7. 40.6 mm to 47.7 mm

### Mixed examination practice 11

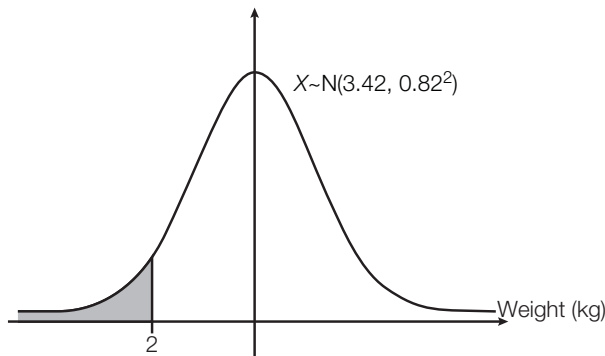
#### Exam-style questions 11

1. (a) (i) 0.68 (ii) 0.95  
 (b) (i) 0.9998 (4 s.f.) (ii) 0.919 (3 s.f.)
2. 0.961 (3 s.f.)
3. (a) 0.150 (3 s.f.) (b) 0.245 (3 s.f.)  
 (c) 0.897 (3 s.f.)
4. (a) 0.933 (3 s.f.) (b) 0.988 (3 s.f.)  
 (c) 0.598 (3 s.f.)

5. (a) 0.141 (3 s.f.)  
 (b) 69.8 m (3 s.f.)  
 (c) 17 (not including Yurek)
6. (a) (i) 0.997 (3 s.f.)      (ii) 0.932 (3 s.f.)  
 (b) 76.8 (3 s.f.)
7. (a) 0.994 (3 s.f.)  
 (b) (i) 0.988 (3 s.f.)      (ii) 0.0124 (3 s.f.)  
 (c) Fewer; with smaller  $\sigma$  the distribution is narrower, so the area under the curve to the left of 230 will be smaller.

### Past paper questions 11

1. (a)  $a = 5.1$  m,  $b = 5.2$  m,  $c = 4.7$  m  
 (b) 0.933 (3 s.f.)  
 (c) 0.234 m (3 s.f.)
2. (a) (i) 68%    (ii) 102  
 (b) (i)



- (ii)  $-1.73$  (3 s.f.)      (iii) 4.17% (3 s.f.)  
 (c) 91.2% (3 s.f.)  
 (d) 5.03 (3 s.f.)

## Chapter 12

### Exercise 12.1

1.

|     | Independent variable          | Dependent variable       | Correlation |
|-----|-------------------------------|--------------------------|-------------|
| (a) | Amount of alcohol consumed    | Reaction time            | Yes         |
| (b) | Number of people in household | Monthly food expenditure | Yes         |
| (c) | Hours of exercise per week    | Body mass                | Yes         |

|     |                         |                          |     |
|-----|-------------------------|--------------------------|-----|
| (d) | Time spent exercising   | Blood sugar level        | Yes |
| (e) | Car mileage             | Value of second-hand car | Yes |
| (f) | Length of middle finger | Sprint time              | No  |
| (g) | Screen size of TV set   | Price of TV set          | Yes |

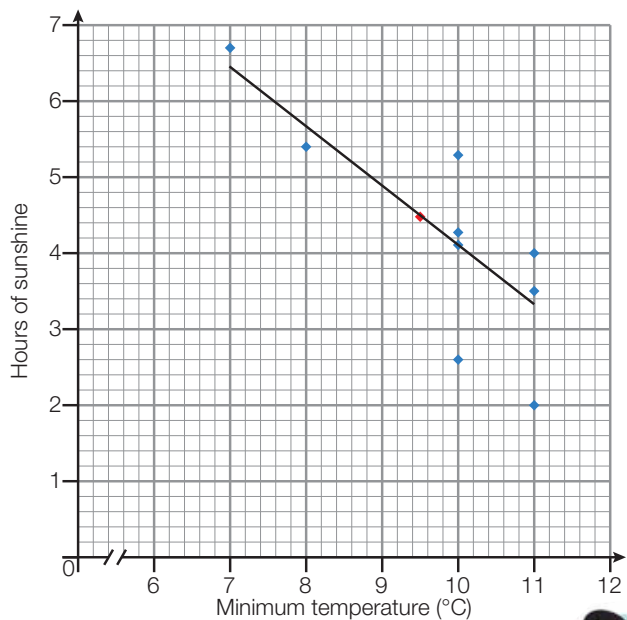
### Exercise 12.2

1. (i) (ii) (iii)

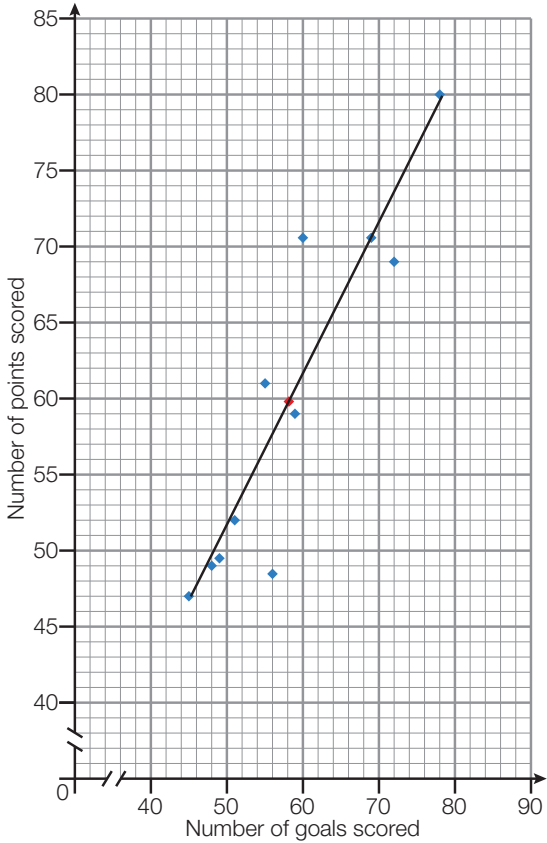
|     | Type of correlation | Independent variable     | Dependent variable      |
|-----|---------------------|--------------------------|-------------------------|
| (a) | Positive            | Height of student        | Arm span                |
| (b) | Negative            | Age of car               | Price of car            |
| (c) | Positive            | Mock exam score          | Final exam score        |
| (d) | Positive            | Hours of sunshine        | Maximum temperature     |
| (e) | Negative            | Number of goals conceded | Number of points scored |

### Exercise 12.3

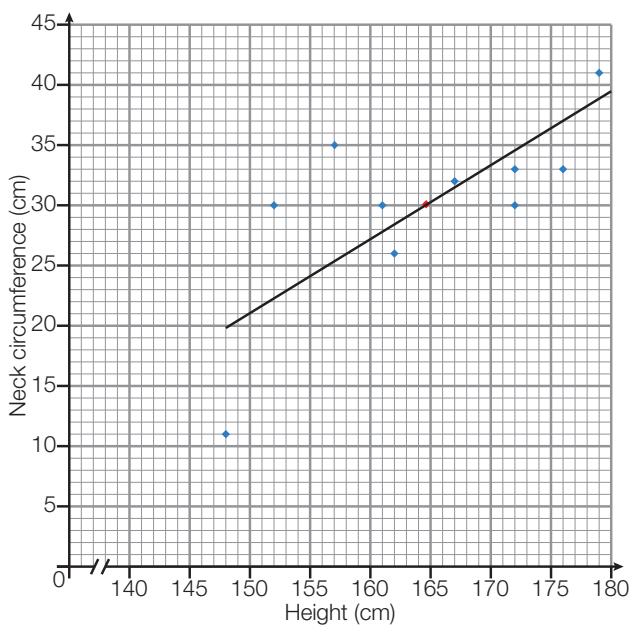
1. (a)



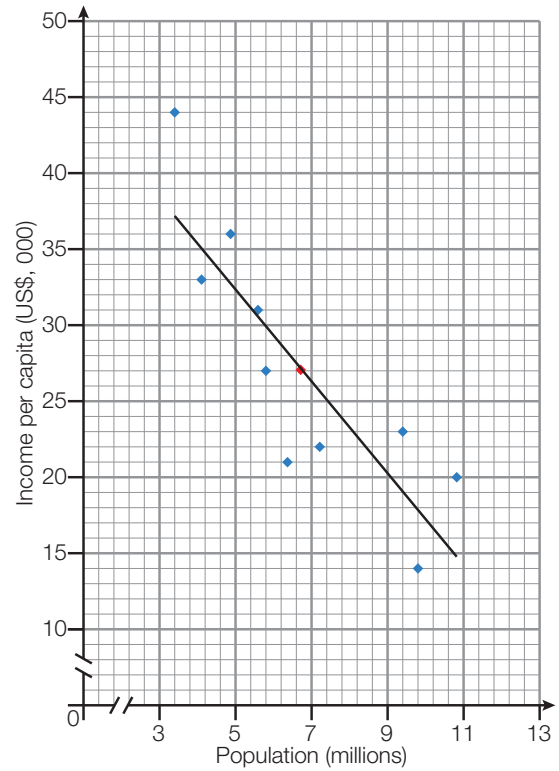
(b)



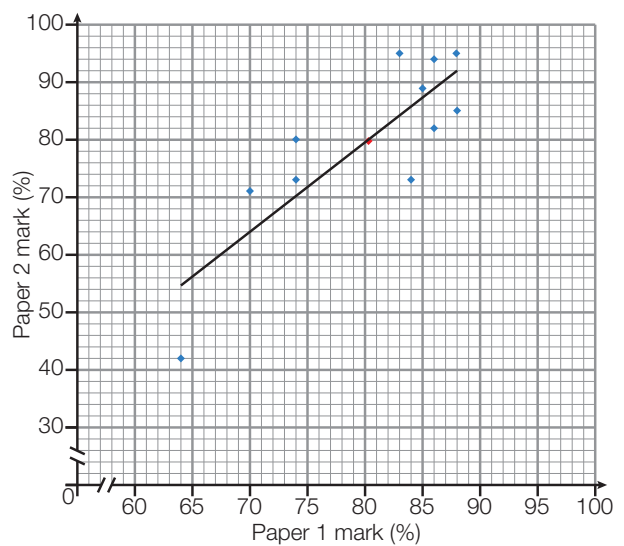
(c)



(d)



(e)



### Exercise 12.4

1. (a)  $-0.912$  (3 s.f.)      (b)  $-0.00769$  (3 s.f.)  
 (c)  $-0.863$  (3 s.f.)      (d)  $0.894$  (3 s.f.)  
 (e)  $0.912$  (3 s.f.)

### Exercise 12.5

1. (a)  $y = 1.0524x - 10.795$   
 (b)  $0.839$  (3 s.f.)  
 (c) Strong positive correlation

2. (a)  $y = 0.119439x + 5.9086$   
 (b) 0.586  
 (c) Moderate positive correlation
3. (a)  $-0.855$  (3 s.f.)  
 (b) Strong negative correlation  
 (c)  $y = -57.984x + 863.72$  (d) US\$545
4. (a)  $y = 0.12354x + 3.8279$  (b) 24.2 cm (3 s.f.)  
 (c) 0.905 (3 s.f.)  
 (d) Strong positive correlation
5. (a) 0.941 (3 s.f.) (b)  $m = 0.647$  (3 s.f.)  
 $c = 1.61$  (3 s.f.)  
 (c) 12.0% (3 s.f.)

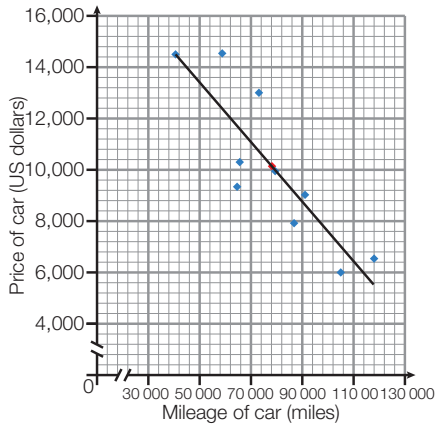
### Exercise 12.6

1. (a) 0.708 (3 s.f.)  
 (b) Positive correlation (moderate)  
 (c) 85% (d) 58%
2. (a)  $y = -0.06706x + 11.5061$   
 (b) \$70,200 (3 s.f.)  
 (c) 9.19% (3 s.f.)  
 (d)  $-0.181$  (3 s.f.)  
 (e) Weak negative correlation, so estimates from the regression line are unreliable.
3. (a)  $r = 0.867$  (3 s.f.); Jessica's assertion is correct.  
 (b)  $m = 0.705$  (3 s.f.),  $c = 14.2$  (3 s.f.)  
 (c) 35 points  
 (d) Reliable as  $r$  is close to 1, indicating that the correlation is strong.

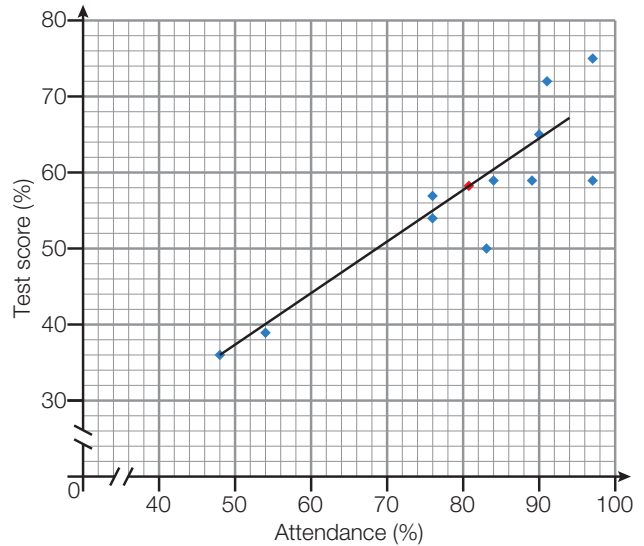
### Mixed examination practice 12

#### Exam-style questions 12

1. (a) Independent variable: mileage; dependent variable: price  
 (b) Negative correlation (strong:  $r = -0.870$ )  
 (c) 78 400 miles, \$10,105  
 (d)



2. (a)



- (b) Positive correlation  
 (c)  $r = 0.887$  (3 s.f.); strong positive correlation

3. (a)  $y = 1.025x - 6.912$   
 (b) 0.744 (3 s.f.)  
 (c) Positive correlation (moderate)
4. (a) 0.992 (3 s.f.)  
 (b) Very strong positive correlation  
 (c)  $y = 4.592x + 14.748$   
 (d) 30.8 amperes (3 s.f.)  
 (e) Not reliable; 18.5 is outside the range of  $x$  values in the data (extrapolation).

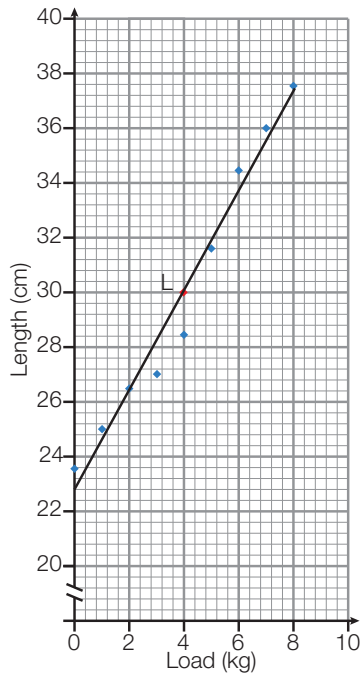
5. (a)  $-0.750$  (3 s.f.)  
 (b)  $m = -0.7245$ ,  $c = 13.9074$   
 (c) Unsuitable, as 15 is outside the range of  $x$  values in the data (extrapolation).  
 (d)  $10.3^{\circ}\text{C}$  (3 s.f.)
6. (a)  $r = 0.959$  (3 s.f.), indicating very strong positive correlation, so Mr Lawrence is right.  
 (b)  $y = 0.700x - 79.609$   
 (c) 1015 pence  
 (d) 3399 pence

7. (a) 0.820 (3 s.f.)  
 (b) Strong positive correlation  
 (c)  $y = 1.844x - 0.04663$   
 (d) (i) 27.8 s (3 s.f.) (ii) 14.2 s (3 s.f.)



### Past paper questions 12

1. (a), (c), (f) (Here, to save space, 1 cm represents 2 kg on the horizontal axis.)



- (b) (i) 4 kg (ii) 2.85 kg (3 s.f.)  
 (iii) 30 cm (iv) 4.78 cm (3 s.f.)  
 (d) (i) 0.986 (3 s.f.)  
 (ii) Very strong positive correlation  
 (e)  $y = 1.825x + 22.7$   
 (g) (i) 32.6 cm (3 s.f.)  
 (ii) Not reliable, as 30 lies far outside the range of  $x$  values in the data.
2. (a)  $y = -0.134x + 20.9$   
 (b) 17  
 (c)  $-0.756$  (3 s.f.)  
 (d) Moderately strong negative correlation

## Chapter 13

### Exercise 13.1

1. (a) 1 (b) 2 (c) 6 (d) 3

### Exercise 13.2

1. (a)

|       | $B_1$ | $B_2$ |
|-------|-------|-------|
| $A_1$ | 66.3  | 131.7 |
| $A_2$ | 88.7  | 176.3 |

- (b)

|       | $B_1$ | $B_2$ | $B_3$ |
|-------|-------|-------|-------|
| $A_1$ | 48.3  | 23.6  | 14.1  |
| $A_2$ | 33.7  | 16.4  | 9.86  |

- (c)

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|-------|-------|-------|-------|-------|
| $A_1$ | 38.3  | 88.5  | 78.0  | 26.1  |
| $A_2$ | 33.8  | 78.2  | 68.9  | 23.1  |
| $A_3$ | 37.8  | 87.3  | 77.0  | 25.8  |

- (d)

|       | $B_1$ | $B_2$ |
|-------|-------|-------|
| $A_1$ | 28.3  | 30.7  |
| $A_2$ | 42.7  | 46.3  |
| $A_3$ | 47.0  | 51.0  |

### Exercise 13.3

1. (a) (i) 1 (ii) 0.207 (3 s.f.)  
 (iii) 0.649 (3 s.f.)  
 (b) (i) 2 (ii) 0.0277 (3 s.f.)  
 (iii) 0.986 (3 s.f.)  
 (c) (i) 6 (ii) 52.9 (3 s.f.)  
 (iii)  $1.25 \times 10^{-9}$  (3 s.f.)  
 (d) (i) 3 (ii) 10.1 (3 s.f.)  
 (iii) 0.0177 (3 s.f.)
2. (a) 12.9 (3 s.f.) (b) 26.7 (3 s.f.)  
 (c) 23.9 (3 s.f.) (d) 4.19 (3 s.f.)
3. (a)  $H_0$ : high performance at GCSE is independent of gender.  
 (b) 1  
 (c) 0.226 (3 s.f.)  
 (d) Do not reject  $H_0$ , as  $0.226 < 3.84$ .
4. (a)  $H_0$ : the age of drivers involved in accidents is independent of gender.  
 (b) 1  
 (c) 4.69 (3 s.f.)  
 (d) Reject  $H_0$ , as  $4.69 > 4.61$ ; there is dependence between age and gender of drivers involved in accidents.
5.  $H_0$ : the genre of books borrowed by readers is independent of age.  $\chi^2_{\text{calc}} = 30.0$ . Reject  $H_0$ , as  $30.0 > 9.49$ .

## Mixed examination practice 13

### Exam-style questions 13

1.  $H_0$ : cell phone ownership is independent of age;  
 $H_1$ : cell phone ownership is dependent on age.  
 $df = 2$ . Expected frequencies:

|                       | 18–34 | 35–54 | 55+   |
|-----------------------|-------|-------|-------|
| Own cell phone        | 367.3 | 530.6 | 839.1 |
| Do not own cell phone | 100.7 | 145.4 | 229.9 |

$$\chi^2_{\text{calc}} = 194.4, \chi^2_{5\%} = 5.99; \chi^2_{\text{calc}} > \chi^2_{5\%}, \text{ so reject } H_0.$$

2. (a) Voting behaviour is independent of the type of work voters do.  
 (b) 2  
 (c) 14.8 (3 s.f.)  
 (d) Reject  $H_0$  and conclude that voting behaviour is dependent on the type of work voters do, because  $\chi^2_{\text{calc}} > \chi^2_{10\%}$ .
3. (a) Low performance in GCSE Mathematics is independent of gender.  
 (b) 2  
 (c) 0.0903 (3 s.f.)  
 (d) Mrs Elwood will not reject  $H_0$ , because  $\chi^2_{\text{calc}} < \chi^2_{5\%}$ .
4. (a) Ownership of smart phones is independent of age.  
 (b) 2  
 (c) 135 (3 s.f.)  
 (d) Reject  $H_0$ , since  $\chi^2_{\text{calc}} > \chi^2_{5\%}$ .
5.  $H_0$ : voting behaviour is independent of the voter's age.  
 $\chi^2_{\text{calc}} = 120.1 > \chi^2_{5\%}$ , so reject  $H_0$  and conclude that voting behaviour is dependent on age.
6. (a) High performance in IB Mathematics is independent of the level at which the subject is studied.  
 (b) Level 5: 8.58, Level 6: 17.68, Level 7: 25.74  
 (c)  $H_0$  should not be rejected, since  $p$ -value (8.06%)  $>$  5%.
7.  $H_0$ : involvement in accidents is independent of the driver's age.  
 $p = 0.00274, \chi^2_{\text{calc}} = 16.2$ . Reject  $H_0$  because  $p = 0.274\% <$  5% and, to confirm the conclusion,  $\chi^2_{\text{calc}} > \chi^2_{5\%} = 7.78$ .

## Past paper questions 13

1. (a)

|         | Drama | Comedy | Film | News |
|---------|-------|--------|------|------|
| Males   | 58    | 119    | 157  | 52   |
| Females | 86    | 98     | 120  | 61   |

- (b)  $H_0$ : favourite TV programme type is independent of gender;  $H_1$ : favourite TV programme type is dependent on gender.  
 (c) 105  
 (d) 12.6 (3 s.f.)  
 (e) (i) 3  
 (ii) 7.815  
 (iii) Reject  $H_0$  and conclude that favourite TV programme type is dependent on gender.
2. (a)  $H_0$ : the size of dog is independent of the time of day.  
 (b) 14.9 (3 s.f.)  
 (d) Reject  $H_0$  and conclude that the size of dog present in the park is dependent on the time of day, because  $\chi^2_{\text{calc}} > \chi^2_{5\%}$ .
3. (a) Favourite type of music is independent of age.  
 (b) 4  
 (c) 51.6 (3 s.f.)  
 (d) Reject  $H_0$ , because  $\chi^2_{\text{calc}} > \chi^2_{5\%} = 9.488$ , or  $p = 1.71 \times 10^{-10} <$  0.05.

## Chapter 14

### Exercise 14.1

1. (a)  $-1$  (b) 4 (c)  $-\frac{11}{8}$   
 (d) 3 (e)  $\frac{19}{4}$
2. (a) AB:  $\frac{1}{2}$  BC:  $-\frac{3}{4}$  CD:  $\frac{1}{2}$  AD:  $-2$   
 (b) AB and CD. They have equal gradients.  
 (c) CD and AD. The product of their gradients is  $-1$ .
3. (a) AE:  $-\frac{2}{3}$  CD:  $\frac{1}{3}$  CE:  $-\frac{2}{5}$  DE:  $-\frac{3}{2}$  DF:  $\frac{3}{2}$  GF:  $\frac{1}{3}$   
 (b) CD and GF with equal gradients.  
 (c) AE and DF. Product of gradients equals  $-1$ .
4. (a) AB:  $\frac{4}{5}$  CD:  $-\frac{14}{5}$  EF: 4 GH:  $\frac{14}{3}$  IJ:  $-\frac{14}{5}$   
 (b) (i) CD and IJ with equal gradients.  
 (ii) None of the lines are perpendicular.  
 No pairing has a product =  $-1$ .

### Exercise 14.2

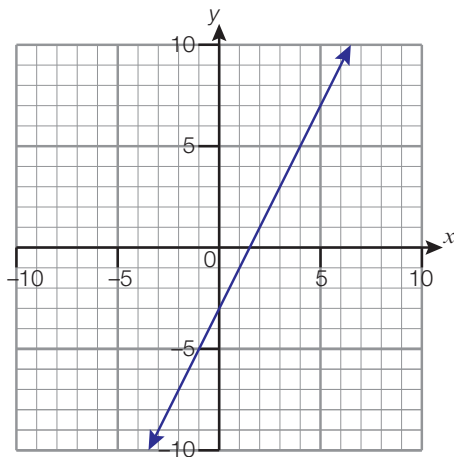
- |                  |                   |
|------------------|-------------------|
| (a) $y = 2x$     | (b) $y = 2x + 1$  |
| (c) $y = 3x - 1$ | (d) $y = 4x - 5$  |
| (e) $y = -3x$    | (f) $y = -3x + 1$ |
| (g) $y = -x + 4$ |                   |
- |                  |                   |
|------------------|-------------------|
| (a) $y = 5$      | (b) $x = 7$       |
| (c) $y = 4x$     | (d) $y = -2x + 3$ |
| (e) $y = 2x - 1$ |                   |
- |                 |                 |                           |
|-----------------|-----------------|---------------------------|
| A: $y = 3x - 1$ | B: $y = -x + 5$ | C: $y = \frac{1}{2}x - 3$ |
|-----------------|-----------------|---------------------------|

### Exercise 14.3

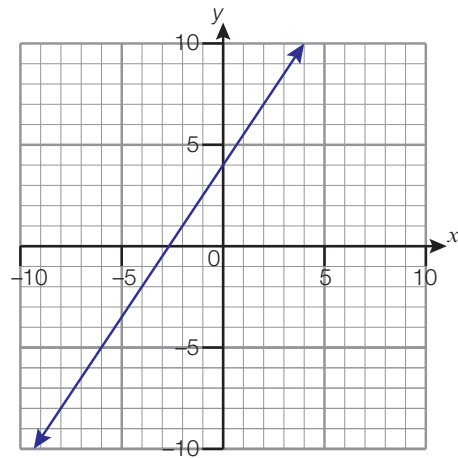
- |                       |                           |
|-----------------------|---------------------------|
| (a) $6x - y - 11 = 0$ | (b) $4x - y + 8 = 0$      |
| (c) $5x + y + 6 = 0$  | (d) $3x + y - 3 = 0$      |
| (e) $4x - 2y - 9 = 0$ | (f) $20x - 28y + 121 = 0$ |
- |                        |                         |
|------------------------|-------------------------|
| (a) $2x + y - 4 = 0$   | (b) $x + 5y - 22 = 0$   |
| (c) $x - 7y - 9 = 0$   | (d) $3x - 24y - 13 = 0$ |
| (e) $7x - 5y + 27 = 0$ | (f) $7x - 2y - 2 = 0$   |
- |                      |                        |
|----------------------|------------------------|
| (a) $2x - y + 1 = 0$ | (b) $2x - 7y - 41 = 0$ |
|----------------------|------------------------|
- |                          |                       |
|--------------------------|-----------------------|
| (a) $7x - y - 23 = 0$    | (b) $5x + 2y + 6 = 0$ |
| (c) $15x + 10y - 47 = 0$ | (d) $5x + 8y = 0$     |
- |                        |                         |
|------------------------|-------------------------|
| (a) $x + 3y - 3 = 0$   | (b) $2x + 7y - 127 = 0$ |
| (c) $3x + 4y - 36 = 0$ | (d) $10x - 15y + 2 = 0$ |
- |                   |                                      |                    |
|-------------------|--------------------------------------|--------------------|
| (a) $\frac{1}{2}$ | (b) $y = \frac{1}{2}x + \frac{5}{2}$ | (c) $y = -2x + 19$ |
|-------------------|--------------------------------------|--------------------|
- |                   |                            |
|-------------------|----------------------------|
| (a) $y = 3x - 5$  | (b) $y = -5x + 10$         |
| (c) $y = -3x + 4$ | (d) $y = \frac{4}{5}x - 2$ |
- |          |                  |                     |
|----------|------------------|---------------------|
| (a) $-1$ | (b) $y = -x - 1$ | (c) $x + y + 1 = 0$ |
|----------|------------------|---------------------|

### Exercise 14.4

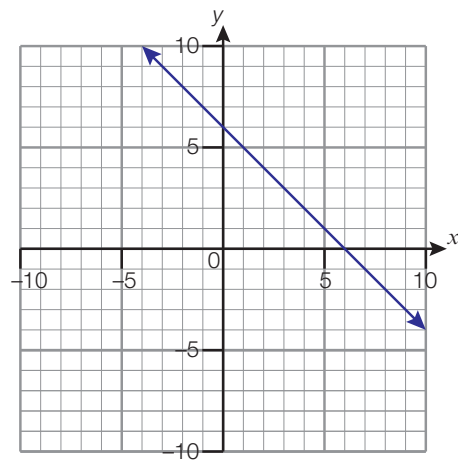
- (a)



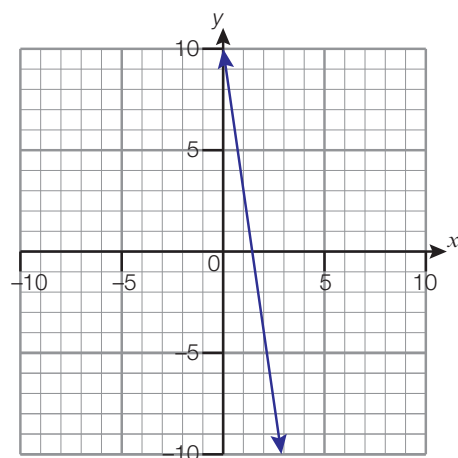
- (b)



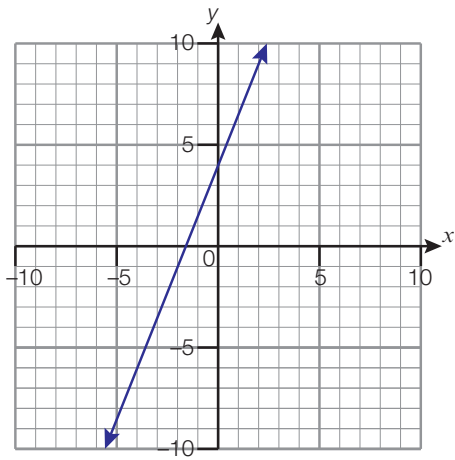
- (c)



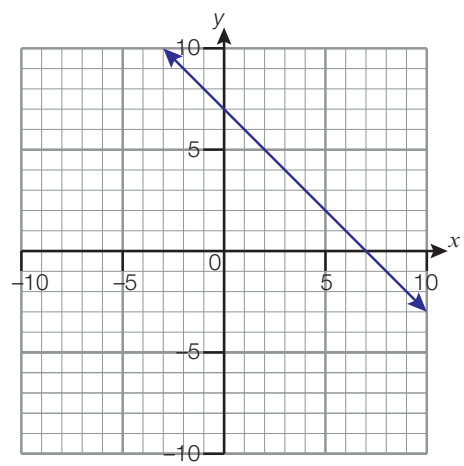
- (d)



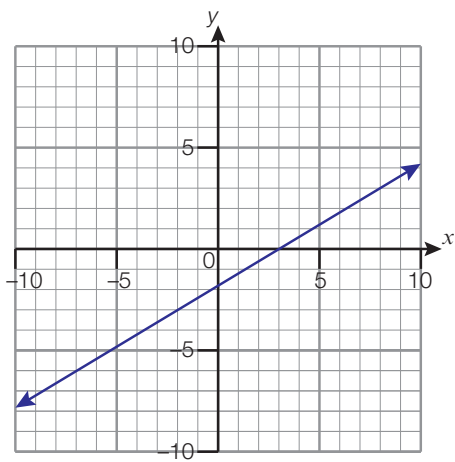
(e)



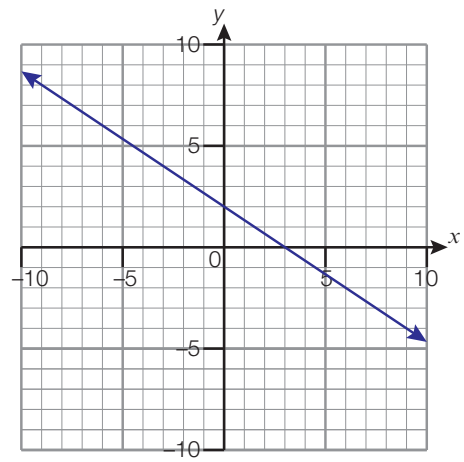
(b)



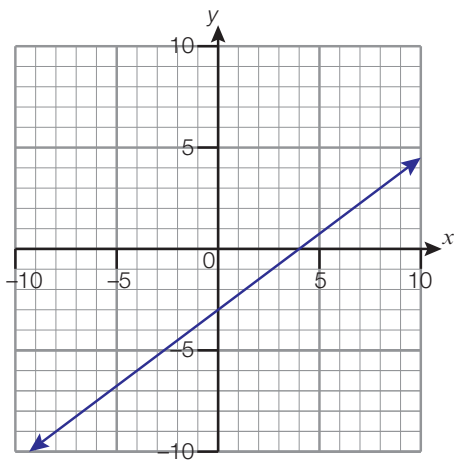
(f)



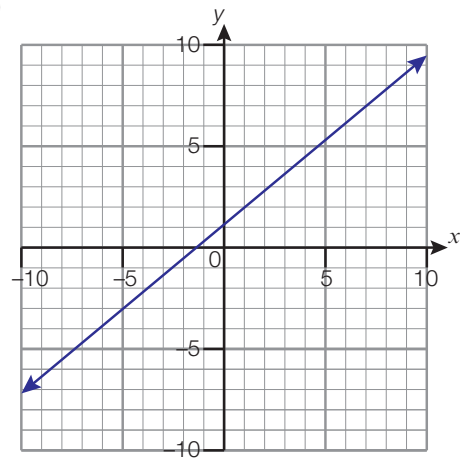
(c)



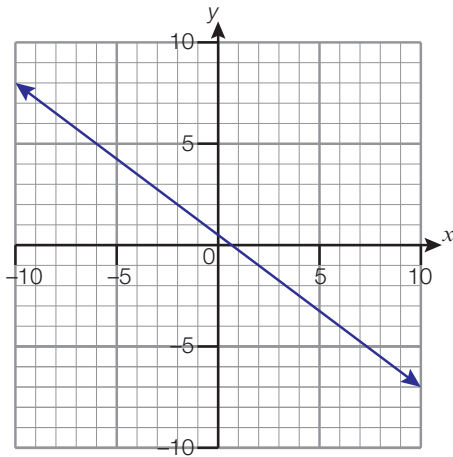
2. (a)



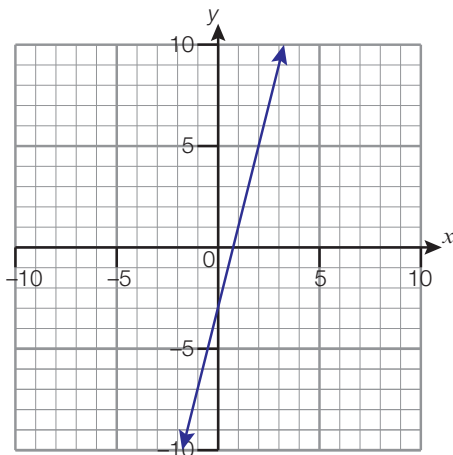
(d)



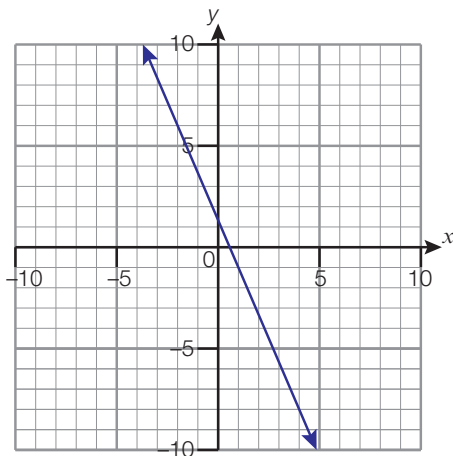
(e)



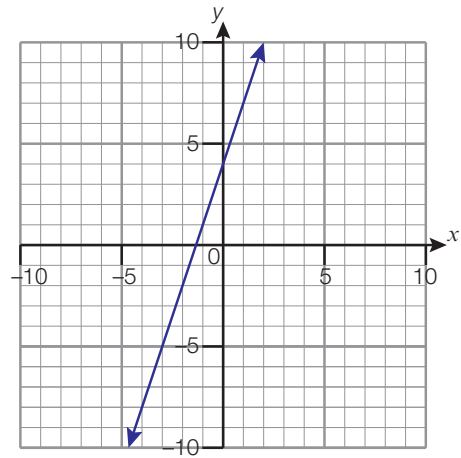
(f)



(g)

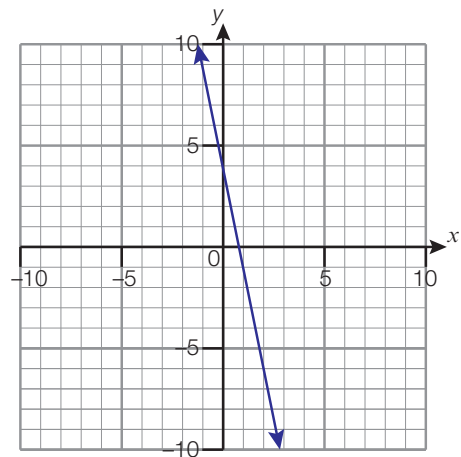


3. (a)



(b)  $y = 3x + 1$

4. (a)



(b)  $y = -5x - 3$

(c)  $x - 5y + 37 = 0$

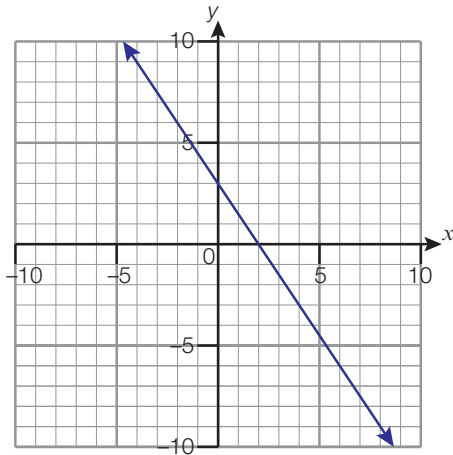
5. (a) (1.143, 4)      (b)  $(\frac{1}{2}, 1)$   
(c) (2, -4)        (d) (-2, 1)  
(e) (4, 1)          (f) (1.288, 1.475)

### Mixed examination practice 14

#### Exam-style questions 14

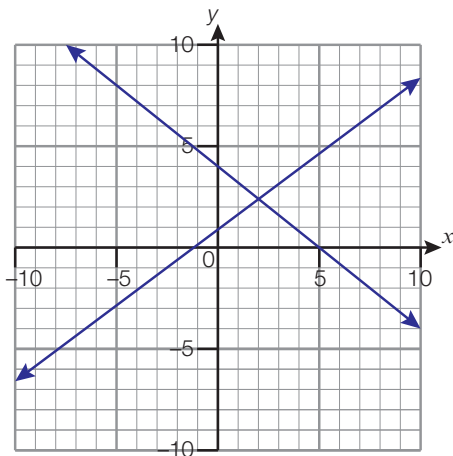
1. (a)  $2x - y + 13 = 0$   
(b)  $5x + y + 8 = 0$   
(c)  $3x + 4y - 19 = 0$
2. Line A:  $y = 3x + 8$   
Line B:  $y = \frac{3}{2}x$   
Line C:  $y = -5x + 6$   
Line D:  $y = \frac{1}{2}x - 7$
3. (a) 5      (b)  $y = 5x - 3$

4. (a)



- (b)  $2x - 3y + 6 = 0$   
 (c)  $3x + 2y + 2 = 0$

5. (a), (c)



- (b) (i) Any line of the form  $y = -\frac{4}{5}x + c$   
 (ii) any line of the form  $y = \frac{5}{4}x + c$   
 (d) (2, 2.4)  
 (e)  $x = 2$  and  $y = 2.4$

### Past paper questions 14

1. (a) 3 (b)  $-\frac{1}{3}$   
 (c)  $y = -\frac{1}{3}x + 9$  (d) (1.5, 8.5)
2. (a)  $-\frac{2}{3}$  (b) (4, 3.5)  
 (c)  $8.06(\sqrt{65})$  (d)  $7x + 4y - 42 = 0$
- (e) Gradient of BC is  $\frac{3}{2}$ , so product of gradients is  $-1$ . Yes, they are perpendicular.

## Chapter 15

### Exercise 15.1

1. (a) 50.2 cm (b) 64.8 cm (c) 79.7 cm  
 (d) 65.3 cm (e) 70.9 mm (f) 6.45 m  
 (g) 10.6 m (h) 11.50 m (i) 67.7 m  
 (j) 64.3 m (k) 65.5 m (l) 81.0 m
2. (a)  $51.3^\circ$  (b)  $47.8^\circ$  (c)  $47.2^\circ$   
 (d)  $42.5^\circ$  (e)  $57.0^\circ$  (f)  $63.4^\circ$
3. 26.0 cm  
 4. 3.53 m  
 5.  $38.0^\circ$   
 6.  $49.1^\circ$

### Exercise 15.2

1. 1030 m  
 2. 765 m  
 3. (a) 71.2 m (b) 426 m

### Exercise 15.3

1. (a) 40.0 cm (b) 40.0 cm (c)  $115^\circ$   
 2. (a) 147 m (b)  $35.9^\circ$   
 3. 72.5 m  
 4.  $28.1^\circ$   
 5. 132 m

### Exercise 15.4

1. (a)  $53.6^\circ$  (b) 35.4 (c) 51.4  
 (d)  $28.4^\circ$  (e) 151 (f)  $52.2^\circ$
2. (a) 73.7 (b) 121 (c) 171
3. (a)  $44.8^\circ$  (b)  $45.5^\circ$  (c)  $64.6^\circ$
4. (a)  $32.0^\circ$  (b)  $54.9^\circ$  (c)  $43.7^\circ$   
 (d)  $36.9^\circ$  (e)  $69.6^\circ$
5. (a) 36.4 cm (b) 72.0 mm (c) 53.1 m  
 (d) 23.9 m (e) 40.3 km

6. 12.2 cm  
 7. 32.4 m

### Exercise 15.5

1. (a) 164 cm (b) 136 cm (c) 71.8 cm  
 2. (a)  $55.2^\circ$  (b)  $19.6^\circ$  (c)  $55.0^\circ$

3. (a) 176 m;  $A = 43.7^\circ$ ;  $C = 31.3^\circ$   
 (b) 51.7;  $P = 27.6^\circ$ ;  
 $Q = 128.4^\circ$  (1 d.p.)  
 (c)  $P = 52.4^\circ$ ;  $Q = 104.8^\circ$ ;  
 $R = 22.8^\circ$  (1 d.p.)  
 (d)  $A = 35^\circ$ ;  $B = 67.1^\circ$ ;  
 $C = 77.9^\circ$
4. 417 m  
 5. 31.1 km

### Exercise 15.6

1. (a)  $6470 \text{ cm}^2$  (b)  $2880 \text{ mm}^2$   
 (c)  $4290 \text{ m}^2$  (d)  $1590 \text{ cm}^2$   
 (e)  $490 \text{ m}^2$

### Exercise 15.7

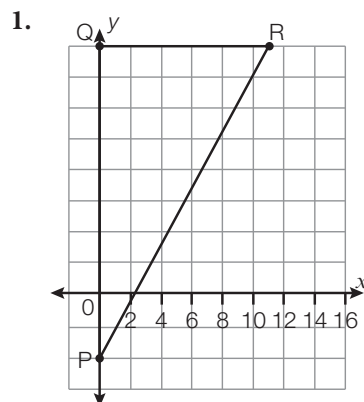
1. (a) 396 m (b) 34 200  $\text{m}^2$   
 (c) 45 800  $\text{m}^2$  (d) 1160 m
2. 3570 m  
 3. 64.2 m  
 4.  $A = 39.9^\circ$ ;  $B = 105.3^\circ$ ;  
 $C = 34.8^\circ$  (1 d.p.)  
 5. 331 km  
 6. 17.4 km

### Mixed examination practice 15

#### Exam-style questions 15

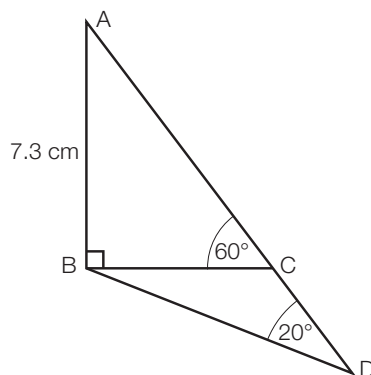
1. 104 m  
 2. 180 m and 73.7 m
3. (a) 2510 m  
 (b)  $157^\circ$  anticlockwise
4. 199 m  
 5.  $56.6^\circ$   
 6.  $319^\circ$   
 7.  $47.9 \text{ m}^2$
8. (a) 181 m (b)  $75.3^\circ$   
 (c) 86.5 m, 477 m  
 (d)  $11\,600 \text{ m}^2$
9. 15.6 m  
 10. 25.7 m

### Past paper questions 15



- (a) Point R on diagram  
 (b)  $28.8^\circ$  ( $28^\circ 49'$ )  
 (c) 43.4 square units

2. (a) (i)



- (ii) 4.21 cm  
 (b) (i) ACD is a straight line.  
 B joins D at  $20^\circ$  to  
 ACD. D must be on  
 AC extended.

(ii)  $\hat{C}BD = 40^\circ$

3. (a) (i)  $73.5^\circ$  (ii) 55.8 m  
 (b) 55.0 m  
 (c) 217 m

## Chapter 16

### Exercise 16.1

1. 56.6 cm
2. (a) 4.72 m (b) 5.49 m
3. 28.3 cm

4. (a) 15 cm  
 (b) 7 cm  
 (c) 13.3 cm
5. (a) 28.3 cm (b) 26.5 cm
6. 41.6 cm
7. (a) 5.83 m  
 (b) 5.22 m  
 (c) AH by 0.728 m  
 (AH = 6.892, AG = 6.164)

### Exercise 16.2

1. (a) 56.6 cm (b)  $38.9^\circ$
2. (a) 14.4 cm (b) 16.5 cm  
 (c)  $29.0^\circ$
3.  $61.1^\circ$
4. (a) 85.4 cm (b) 89.0 cm  
 (c)  $16.3^\circ$
5.  $x = 19.4^\circ$   $y = 127^\circ$
6. (a) (i) 17.3 cm  
 (ii) 7.07 cm  
 (b)  $78.2^\circ$
7. (a) ME = MF = 87.7 cm  
 (b)  $54.3^\circ$
8. (a) 107 cm (b)  $20.3^\circ$   
 (c)  $20.1^\circ$

### Exercise 16.3

1.

|   | (a) Total surface area | (b) Volume           |
|---|------------------------|----------------------|
| 1 | $5027 \text{ cm}^2$    | $23695 \text{ cm}^3$ |
| 2 | $56 \text{ m}^2$       | $28 \text{ m}^3$     |
| 3 | $1504 \text{ cm}^2$    | $3840 \text{ cm}^3$  |
| 4 | $1414 \text{ cm}^2$    | $7069 \text{ cm}^3$  |
| 5 | $1531 \text{ cm}^2$    | $3528 \text{ cm}^3$  |
| 6 | $6082 \text{ cm}^2$    | $44602 \text{ cm}^3$ |

2. (a)  $33\,510 \text{ cm}^3$   
 (b)  $28\,953 \text{ cm}^3$   
 (c)  $386\,039 \text{ cm}^3$   
 (d)  $157\,080 \text{ cm}^3$   
 (e)  $837\,758 \text{ cm}^3$

3. (a)  $5027 \text{ cm}^2$  (b)  $5429 \text{ cm}^2$   
 (c)  $30\,561 \text{ cm}^2$  (d)  $16\,493 \text{ cm}^2$   
 (e)  $50\,265 \text{ cm}^2$
4. (a)  $r = 6.20 \text{ cm}$  (b)  $x = 10 \text{ cm}$
5. (a)  $29.4 \text{ m}^3$  (b)  $16 \text{ m}^2$
6. (a) radius =  $3.2 \text{ cm}$ , length =  $25.6 \text{ cm}$   
 (b)  $275 \text{ cm}^3$

### Mixed examination practice 16

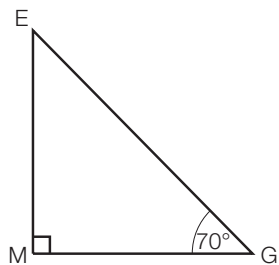
#### Exam-style questions 16

1. (a)  $11.3 \text{ cm}$  (b)  $13.9 \text{ cm}$
2. (a)  $71.1^\circ$  (b)  $17.7^\circ$
3.  $5.10 \text{ m}$
4. (a)  $115\,000 \text{ cm}^3$  (b)  $12\,700 \text{ cm}^2$
5.  $2090 \text{ cm}^3$
6.  $25.5 \text{ cm}$  and  $76.4 \text{ cm}$
7. (a) (i)  $7240 \text{ cm}^3$   
 (ii)  $1810 \text{ cm}^2$   
 (b)  $1810 \text{ cm}^3$   
 (c)  $7.56 \text{ cm}$   
 (d) No, total surface area of 4 smaller spheres ( $2872 \text{ cm}^2$ ) is more than the surface area of original sphere.

#### Past paper questions 16

1. (a)  $1294.14 \text{ cm}^3$  (2 d.p.)  
 (b) 6  
 (c) (i)  $431 \text{ cm}^3$   
 (ii)  $0.000431 \text{ m}^3$  or  $4.31 \times 10^{-4} \text{ m}^3$

2. (a)



- (c) (i)  $EG = 14.6 \text{ cm}$   
 (ii)  $= 37.8^\circ$   
 (d)  $392 \text{ cm}^2$   
 (e)  $458 \text{ cm}^3$

## Chapter 17

### Exercise 17.1

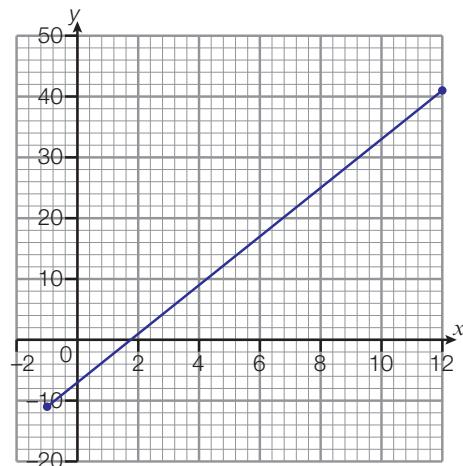
1. (a) No (b) Yes (c) Yes  
 (d) No (e) Yes (f) Yes

### Exercise 17.2

1. (a) 4 (b)  $-11$   
 (c)  $-7$  (d)  $3a - 8$
2. (a)  $-19$  (b) 12  
 (c)  $-11$  (d)  $11 - 10a$
3. (a) 27 (b) 5  
 (c)  $-3$  (d)  $2c^2 - 7c + 5$
4. (a)  $-147$  (b)  $-15$  (c) 429
5. (a) 9.1 (b) 1.9  
 (c)  $-10.5$  (d)  $-1550$
6. (a)  $\frac{5}{9}$  (b)  $-\frac{11}{5} = -2.2$   
 (c) 3 (d)  $\frac{3+x}{x-1}$

### Exercise 17.3

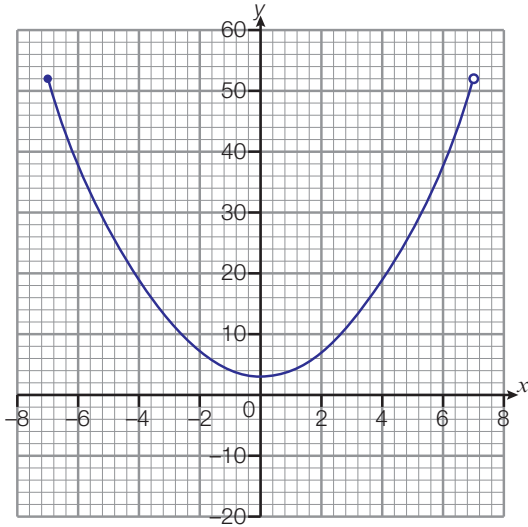
1. (a)  $-4 < x \leq 4$ ,  $-5 \leq y < 11$   
 (b)  $x > -4$ ,  $y \geq -2$   
 (c)  $x \geq -5$ ,  $y \geq -80$   
 (d)  $-3 < x \leq 6$ ,  $3 < y \leq 15$   
 (e)  $x \geq -3$ ,  $y \leq 12.8$   
 (f)  $-6 \leq x \leq 4$ ,  $-5.125 \leq y \leq 50$
2. (a)



$$-11 \leq y \leq 41$$

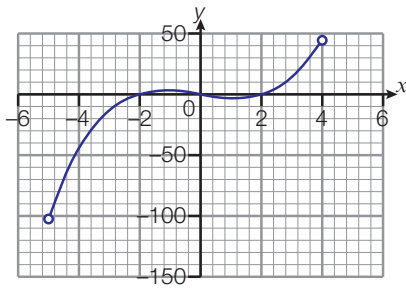


(b)



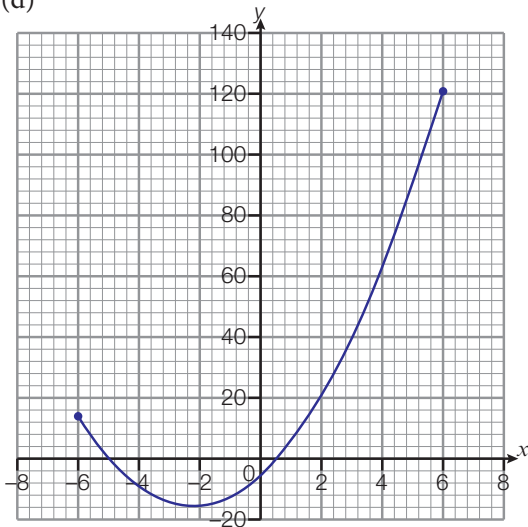
$$3 \leq y < 52$$

(c)



$$-105 \leq y \leq 48$$

(d)



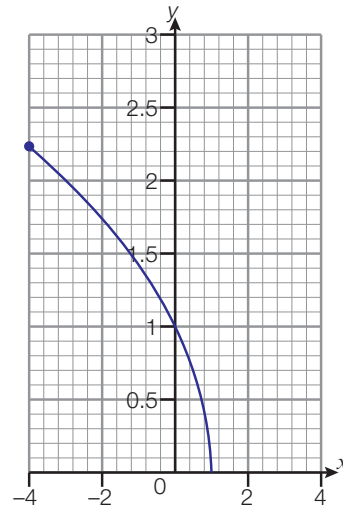
$$-15.125 < y < 121$$

### Exercise 17.4

- (a) (i)  $x = 0$       (ii)  $y = 0$   
(b) (i)  $x = 0$       (ii)  $y = 0$   
(c) (i)  $x = -1$      (ii)  $y = 0$   
(d) (i)  $x = -1$      (ii)  $y = 0$   
(e) (i)  $x = -0.5$    (ii)  $y = 0$   
(f) (i)  $x = 1.5$     (ii)  $y = 0$   
(g) (i)  $x = 1.5$     (ii)  $y = 5$   
(h) (i)  $x = \frac{2}{3}$         (ii)  $y = 4$

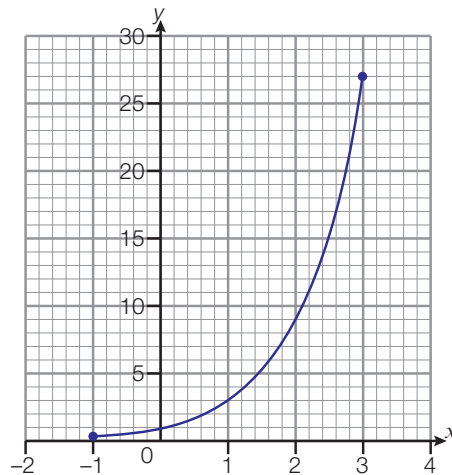
### Exercise 17.5

1. (a)



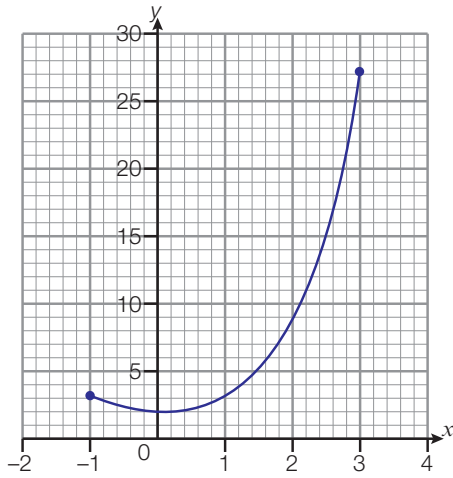
- (b)  $0 \leq f(x) \leq 2.24$ ; the curve does not exist beyond  $x = 1$ , and lies above the  $x$ -axis for all values of  $x < 1$ .  
(c)  $5 \leq g(x) \leq 7.24$ ; the graph of  $g(x)$  is obtained by shifting the graph of  $f(x)$  up by 5 units.

2. (a)



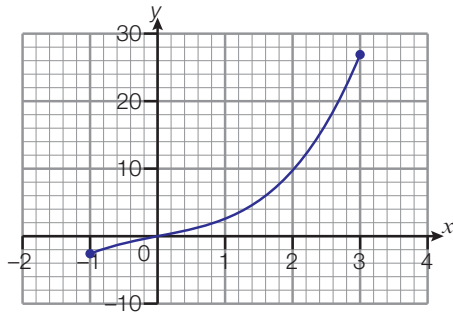
$$\frac{1}{3} \leq f(x) \leq 27$$

(b)



$$2 \leq g(x) \leq 27.0$$

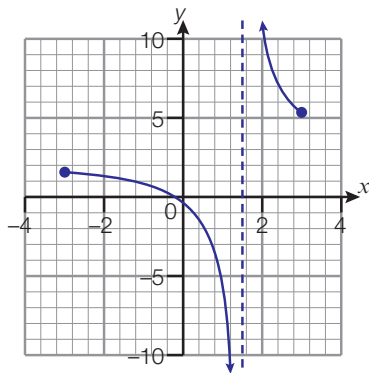
(c)



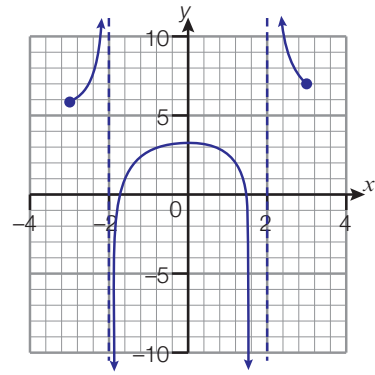
$$-2\frac{2}{3} \leq h(x) \leq 27.0$$

(d)  $g(x) + h(x) = 2f(x)$ , so the range is  $\frac{2}{3} \leq y \leq 54$

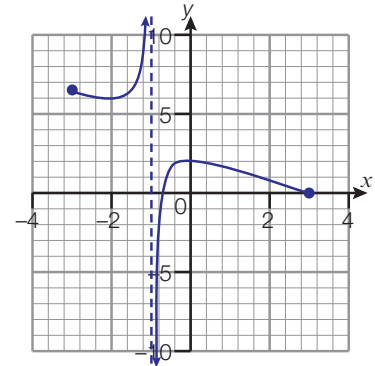
3. (a)



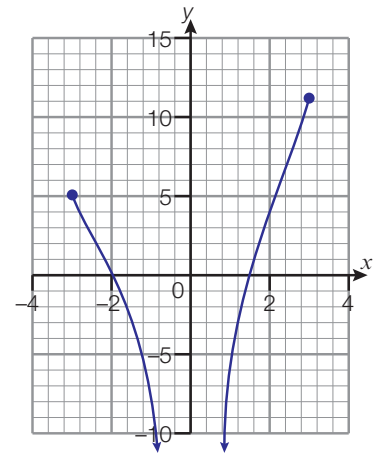
(b)



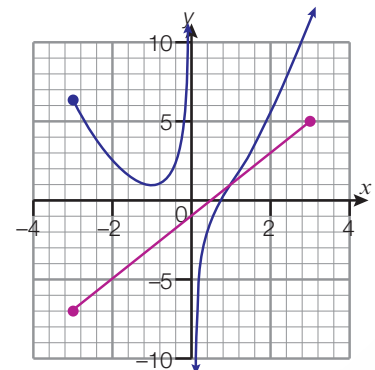
(c)



(d)

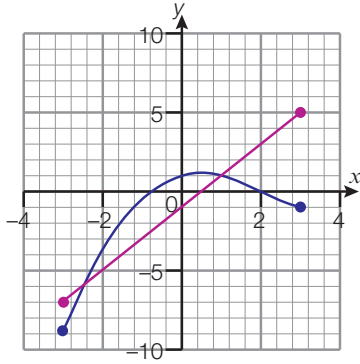


4. (a)



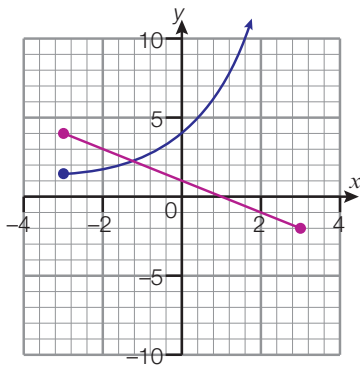
Point of intersection: (1, 1)

(b)



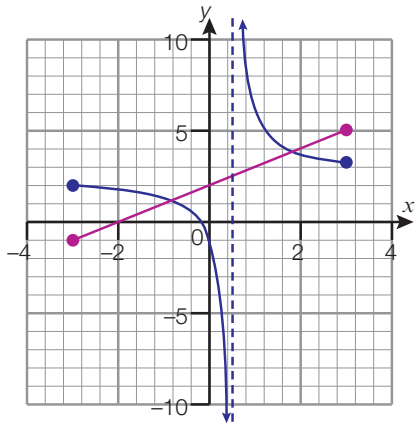
Points of intersection:  $(-2.48, -5.95)$  and  $(1, 1)$

(c)



Point of intersection:  $(-1.26, 2.26)$

(d)



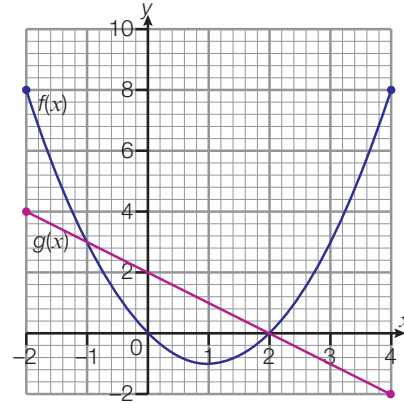
Points of intersection:  $(-0.82, 1.18)$  and  $(1.82, 3.82)$

### Exercise 17.6

1. (a)

|        |    |    |   |    |   |    |    |
|--------|----|----|---|----|---|----|----|
| $x$    | -2 | -1 | 0 | 1  | 2 | 3  | 4  |
| $f(x)$ | 8  | 3  | 0 | -1 | 0 | 3  | 8  |
| $g(x)$ | 4  | 3  | 2 | 1  | 0 | -1 | -2 |

(b)

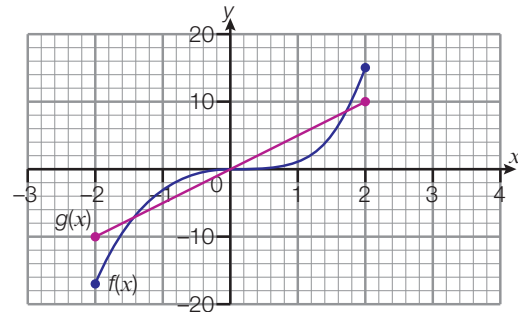


(c)  $-1, 2$

2. (a)

|        |     |    |    |   |    |
|--------|-----|----|----|---|----|
| $x$    | -2  | -1 | 0  | 1 | 2  |
| $f(x)$ | -17 | -3 | -1 | 1 | 15 |
| $g(x)$ | -10 | -5 | 0  | 5 | 10 |

(b)

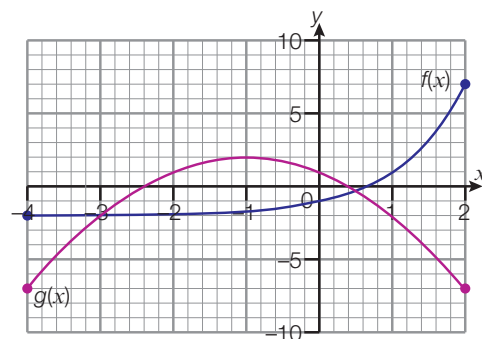


(c)  $-1.5, -0.2, 1.7$

3. (a)

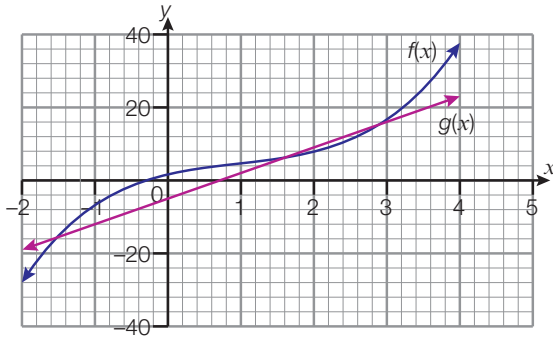
|        |       |       |       |       |    |    |    |
|--------|-------|-------|-------|-------|----|----|----|
| $x$    | -4    | -3    | -2    | -1    | 0  | 1  | 2  |
| $f(x)$ | -1.99 | -1.96 | -1.89 | -1.67 | -1 | 1  | 7  |
| $g(x)$ | -7    | -2    | 1     | 2     | 1  | -2 | -7 |

(b)



(c)  $-3.0, 0.5$

4. (a)



(b) 3      (c) -1.5, 1.7, 2.8

5. (a) (i) 3      (ii) -1.03, -0.207, 1.01  
 (b) (i) 1      (ii) 1.71  
 (c) (i) 1      (ii) 0.330  
 (d) (i) 2      (ii) 0.847, 3.47  
 (e) (i) 3      (ii) -1.73, -1.29, 1.64  
 (f) (i) 3      (ii) -1.79, 1.54, 2

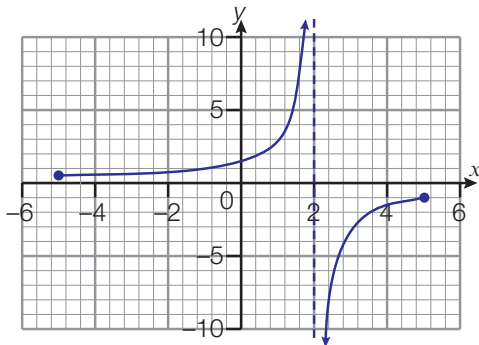
6. (a) -1.37, 4.37      (b) -2.93, -0.363, 0.627  
 (c) -3.25, -1.62, 0.878      (d) -2.24, 2.24  
 (e) 1.17, 2.29      (f) -2.96, -0.633  
 (g) -4.06, -1.12, 2.18

7. (a)  $t = 81.3$       (b)  $x = 0$   
 (c)  $t = 0.928$       (d)  $x = -0.0692$   
 (e)  $x = -3.06, 0.969$       (f)  $x = -2.29, -0.671$   
 (g)  $x = -2.71, -1.06$       (h)  $x = 1.45$

### Mixed examination practice 17

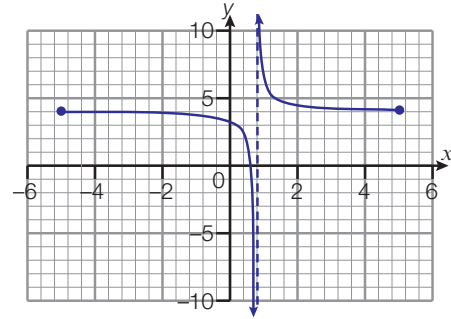
#### Exam-style questions 17

1. (a) -13      (b) 8  
 (c) (i)  $7a - 13$   
 (ii)  $7a - 27$
2. (a)  $x \geq -3$       (b)  $y \leq 13$       (c) -3, 0.25, 2
3. (a)



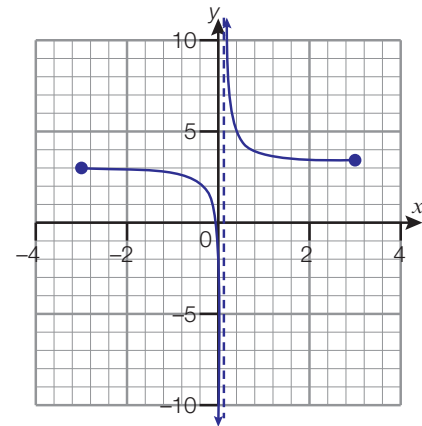
(i)  $x = 2$       (ii)  $y = 0$

(b)



(i)  $x = \frac{7}{9}$       (ii)  $y = 4$

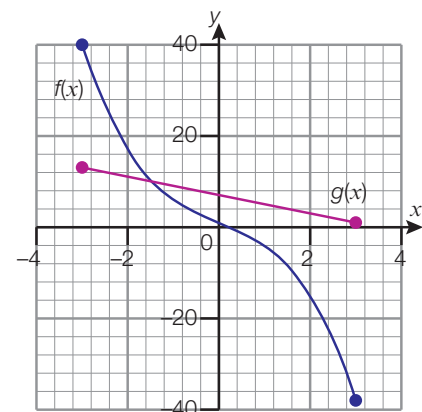
4.



5. (a)

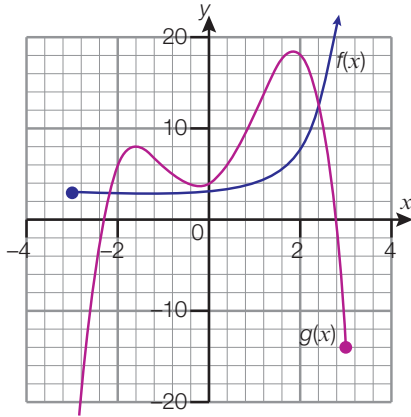
|        |    |    |    |   |    |     |     |
|--------|----|----|----|---|----|-----|-----|
| $x$    | -3 | -2 | -1 | 0 | 1  | 2   | 3   |
| $f(x)$ | 40 | 17 | 6  | 1 | -4 | -15 | -38 |
| $g(x)$ | 13 | 11 | 9  | 7 | 5  | 3   | 1   |

(b)



(c) -1.5

6. (a)



(b) 2

(c)  $-2.2, 2.4$

## Chapter 18

### Exercise 18.1

- (a)  $D = 50 + 0.15m$   
 (b) 400  
 (c) Safe Ride: it costs less (has a lower graph) for  $m > 400$
- (a)  $C(n) = 6.50 + 0.035n$   
 (b) £52  
 (c) 1840 units
- (a)  $C(n) = 5.18 + 0.13n$   
 (b) £70.18  
 (c) 650 units
- (a) 60 visits  
 (b) (i) Gym Buddies  
 (ii) Fit Mates  
 (c) (i) At least £24  
 (ii) At least £30
- (a) (i) 150    (ii) 120    (iii) 100  
 (b) £180  
 (c) £200  
 (d) £160

### Exercise 18.2A

- (a) (i)  $(-1.5, -2.25)$   
 (ii)  $x = -1.5$   
 (iii)  $f(x) \geq -2.25$

- (i)  $(3.5, -10.25)$   
 (ii)  $x = 3.5$   
 (iii)  $f(x) \geq -10.25$
- (i)  $(0.25, -5.875)$   
 (ii)  $x = 0.25$   
 (iii)  $f(x) \leq -5.875$
- (i)  $(0.667, 7.67)$   
 (ii)  $x = 0.667$   
 (iii)  $g(x) \geq 7.67$
- (i)  $(-1, -16)$   
 (ii)  $x = -1$   
 (iii)  $g(x) \geq -16$
- (i)  $(0.5, -1.25)$   
 (ii)  $x = 0.5$   
 (iii)  $g(x) \geq -1.25$
- (i)  $(0.75, 2.22)$   
 (ii)  $x = 0.75$   
 (iii)  $h(x) \geq 2.22$
- (i)  $(3.5, 13.9)$   
 (ii)  $x = 3.5$   
 (iii)  $f(x) \leq 13.9$

- (a)  $(0, 1.44)$   
 (i) Max 1.44    (ii)  $x = 0$     (iii)  $f(x) \leq 1.44$

(b)  $(2.5, 10)$   
 (i) Max 10    (ii)  $x = 2.5$     (iii)  $g(x) \leq 10$

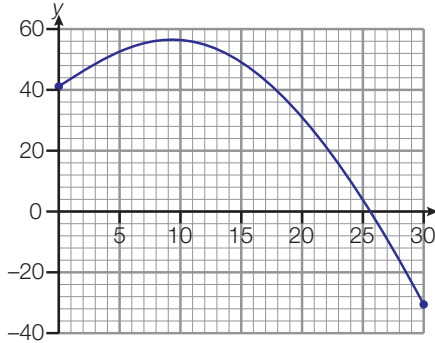
(c)  $(-1.7, 8)$   
 (i) Min 8    (ii)  $x = -1.7$     (iii)  $h(x) \geq 8$

### Exercise 18.2B

- $(-1.83, 0)$  and  $(3.83, 0)$
- (a) (i)  $f(x)$     (ii)  $g(x)$     (iii)  $h(x)$   
 (b)  $f(x): (0, -5); g(x): (0, 5); h(x): (0, \frac{25}{4})$
- (a) (i)  $(-2, 0)$  and  $(2, 0)$     (ii)  $(0, -12)$   
 (b) (i)  $(-1.33, 0)$  and  $(1.33, 0)$     (ii)  $(0, 16)$   
 (c) (i)  $(-1.27, 0)$  and  $(6.27, 0)$     (ii)  $(0, 8)$   
 (d) (i)  $(-2, 0)$  and  $(1.33, 0)$     (ii)  $(0, 8)$   
 (e) (i)  $(-2.92, 0)$  and  $(3.42, 0)$     (ii)  $(0, -20)$   
 (f) (i)  $(1.64, 0)$  and  $(3.36, 0)$     (ii)  $(0, -23.6)$
- (a)  $(0, 0)$  and  $(5, 0)$   
 (b)  $x = 2.5$   
 (c)  $-8.4 \leq f(x) \leq 8.75$
- (a)  $P(-6.82, 0), Q(-2.75, 33.1), R(0, 18), S(1.32, 0)$   
 (b)  $f(x) \leq 33.1$   
 (c)  $x = -2.75$

### Exercise 18.3

- (a) 20.4 m      (b) 21.8 m      (c) 4.08 s
- 43.8, 82.7, 132.6, 193.7, 265.9, 349.1
- (a)



- (b) €9 million  
(c) €53.8 million
- (a) 12 m      (b) 2 s  
(c) 12.25 m      (d) 4.12 s
  - (a) (i) 15.3 m      (ii) 35.3 m  
(b) (i) 13.7 m      (ii) 15.0 m  
(c) 10.8 m

### Mixed examination practice 18

#### Exam-style questions 18

- (a) (i) (0.9, -4.05)      (ii)  $x = 0.9$       (iii)  $y \geq -4.05$   
(b) (i) (0, 7)      (ii)  $x = 0$       (iii)  $y \leq 7$   
(c) (i) (1.5, -30.25)      (ii)  $x = 1.5$       (iii)  $y \geq -30.25$
- (a) Max (-0.333, 11.7)  
(b)  $x = -0.333$   
(c)  $f(x) \leq 11.7$
- (a)  $a = 140, b = 98$       (b) \$630      (c) 7 hours
- (a)  $135 - x$       (c) 45 m by 90 m
- (a) 34.2 m      (b) 240 m      (c) 40 m or 200 m
- (a) (i) £11.60      (ii) £29.00  
(b) Saif: £10.35; Jeevan: £29.20  
(c) (i) MATHSMANAGER  
(ii) At least £3.20

#### Past paper questions 18

- (a) 25      (b) 13.7  
(c) 0.535      (d) 54.0

## Chapter 19

### Exercise 19.1

- (a)  $y = 0$       (b)  $y = 0$       (c)  $y = 5$   
(d)  $y = -4$       (e)  $y = -4$
- (a) (0, 1);  $0.008 \leq g(x) \leq 15625$   
(b) (0, 1);  $4.096 \times 10^{-9} \leq g(x) \leq 2.44 \times 10^8$   
(c) (0, 25);  $0.0016 \leq g(x) \leq 7.63 \times 10^{11}$   
(d) (0, 78125);  $0.008 \leq g(x) \leq 78125$   
(e) (0, 0.0016);  $0.0016 \leq g(x) \leq 3125$
- (a) (i)  $y = 0$       (ii) (0, 1)  
(iii)  $0 < f(x) \leq 16$   
(b) (i)  $y = 0$       (ii) (0, 1)  
(iii)  $0 < f(x) \leq 64$   
(c) (i)  $y = 0$       (ii) (0, 1)  
(iii)  $0 < f(x) \leq 256$   
(d) (i)  $y = 0$       (ii) (0, 1)  
(iii)  $0 < f(x) \leq 8$   
(e) (i)  $y = 5$       (ii) (0, 6)  
(iii)  $5 < f(x) \leq 7781$   
(f) (i)  $y = -4$       (ii) (0, -3)  
(iii)  $-4 < f(x) \leq -3$   
(g) (i)  $y = 0$       (ii) (0, 2)  
(iii)  $0 < f(x) \leq 31250$   
(h) (i)  $y = 0$       (ii) (0, 4)  
(iii)  $0 < f(x) \leq 3.91 \times 10^7$
- (a) (i) 1      (ii) 125  
(iii) 0.2      (iv) 2.24  
(b) (i) 1      (ii) 15625  
(iii) 0.04      (iv) 5  
(c) (i) 125      (ii)  $1.95 \times 10^6$   
(iii) 5      (iv) 625  
(d) (i)  $1.28 \times 10^{-5}$       (ii) 0.2  
(iii)  $5.12 \times 10^{-7}$       (iv)  $6.4 \times 10^{-5}$   
(e) (i) 1.608      (ii) 201  
(iii) 0.3216      (iv) 3.60  
(f) (i) 32640      (ii)  $3.264 \times 10^{10}$   
(iii) 326.4      (iv)  $3.264 \times 10^5$   
(g) (i)  $5.27 \times 10^{-8}$       (ii) 52.7  
(iii)  $5.27 \times 10^{-11}$       (iv)  $1.67 \times 10^{-6}$   
(h) (i) 1      (ii) 83.8  
(iii) 0.229      (iv) 2.09  
(i) (i) 34      (ii) 693  
(iii) 12.4      (iv) 56.2  
(j) (i) 0.0238      (ii) 134  
(iii) 0.00134      (iv) 0.100

### Exercise 19.2

- 52 996
    - 60 876
    - 63 965
  - 11.3 years
    - 37.1 years
  - 35.0 years
    - 55.5 years
- 6.87 million
    - 8.21 million
  - 2031
  - 2007
- 200 mg
  - 117 mg
    - 68.7 mg
  - 12.1 hours
- 420
  - 488
  - 2540
  - 6250
  - 23.1 hours
  - 29.3 hours
- 95.3°C
    - 88.7°C
    - 75.0°C
  - 93.0 minutes
  - 382 minutes

### Exercise 19.3

- (0, 6)
    - (-3, 0), (1, 0), (2, 0)
  - (0, 3)
    - (-3, 0), (1, 0)
  - (0, 8)
    - (-4, 0), (-2, 0), (1, 0)
  - (0, 6)
    - (-3, 0), (-1, 0), (0.4, 0)
  - (0, -4)
    - (-2, 0), (1, 0)
  - (0, 0)
    - (-6, 0), (0, 0), (1, 0)
  - (0, 6)
    - (-3, 0), (0.5, 0), (2, 0)
  - (0, -8)
    - (-2.67, 0), (2.67, 0)
  - (0, 4)
    - (-4, 0), (-1, 0), (0.5, 0), (1, 0)
  - (0, -9)
    - (-3, 0), (-1, 0), (3, 0)
- (0, -12); (-3, 0), (4, 0)
    - $-12.25 \leq f(x) \leq 8$
  - (0, 1); (-0.140, 0), (7.14, 0)
    - $-7 \leq f(x) \leq 13.25$
  - (0, 0); (-2, 0), (0, 0), (4, 0)
    - $-96 \leq f(x) \leq 64$
  - (0, 6); (-2, 0), (1, 0), (3, 0)
    - $-4.06 \leq f(x) \leq 18$
  - (0, 3); (-0.538, 0), (2.33, 0)
    - $-24 \leq g(x) \leq 3.39$
  - (0, 4); (-3.06, 0)
    - $-24 \leq g(x) \leq 226$

- (0, 10); (-1.19, 0), (1.20, 0)
  - $-19.3 \leq f(x) \leq 10.0$
- (0, 6)
  - $5.75 \leq f(x) \leq 20$

### Exercise 19.4

- 1.4115
    - 1.4535
    - 1.4262
  - 1.46

- 

| Decade                |                      | 1970     | 1990     | 2010     |
|-----------------------|----------------------|----------|----------|----------|
| Population (billions) | Actual               | 3.706618 | 5.278640 | 6.848933 |
|                       | Estimated from model | 3.702066 | 5.272624 | 6.854624 |
| Percentage error      |                      | 0.123%   | 0.114%   | 0.0831%  |

- 7.58 billion
  - 8.25 billion
  - 8.85 billion

- 

| Year                         | 2000    | 2002   | 2004   | 2006    |
|------------------------------|---------|--------|--------|---------|
| Price of silver on 1 January | \$15.78 | \$4.68 | \$9.09 | \$18.57 |

- US\$15.34 per ounce
  - 0.11%

- 

| Year                       | 2002  | 2004     | 2006     | 2008   | 2010     | 2012      |
|----------------------------|-------|----------|----------|--------|----------|-----------|
| Price of gold on 1 January | \$278 | \$293.24 | \$387.29 | 538.58 | \$768.80 | \$1116.07 |

- US\$925.52 per ounce
  - 8.43%

### Mixed examination practice 19

#### Exam-style questions 19

- (0, -4)
  - (-1.22, 0), (-0.47, 0), (1.15, 0), (3.04, 0)
  - $-128 \leq g(x) \leq 16.6$
- (0, 1)
  - $y = 0$
  - $f(x) > 0$
- $A = 3$
  - $y = 5$
- 47.3°C
    - 37.8°C
    - 25.7°C
    - 24.6°C
  - 0.112 h (6.73 min)
    - 0.982 h (58.9 min)
    - 1.50 h

5.

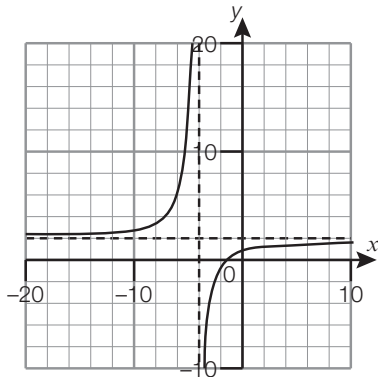
| Year            | 2000 | 2002 | 2004 | 2006 | 2008 | 2010 |
|-----------------|------|------|------|------|------|------|
| Debt (% of GDP) | 30.8 | 28.1 | 31.4 | 34.5 | 37.8 | 48.1 |

6.  $C_0 = 8 \text{ mg/ml}$  and  $k = -0.0851$

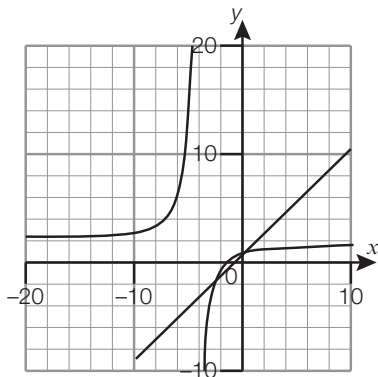
7. (a) (i) 28 040 MW  
 (ii) 46 680 MW  
 (iii) 47 340 MW  
 (iv) 40 090 MW  
 (b) (i) 8.58 a.m. and 9.37 p.m.  
 (ii) 10.32 a.m. and 8 p.m.

### Past paper questions 19

1. (a)



- (b)  $x = -4$   
 (c)



- (d)  $(-2.85078, -2.35078)$  or  $(0.35078, 0.85078)$   
 (e) 1  
 (f)  $y = -x - 5$

2. (a) 1800  
 (b) 145 800  
 (c) 33.5 hours

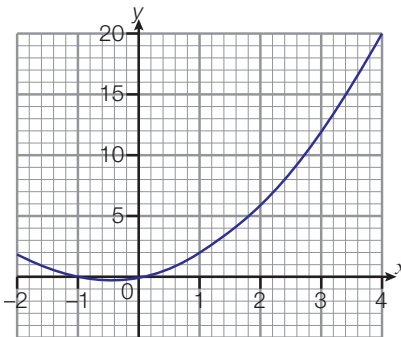
3. (a)  $90^\circ\text{C}$  (b)  $y = 16$   
 (c)  $16^\circ\text{C}$  (d)  $25.4^\circ\text{C}$   
 (e)  $42.8^\circ\text{C}$   
 (f) 1.55 minutes or 93 seconds

## Chapter 20

### Learning links 20A

1. 15.25, 13.89, 12.61, 12.0601,  
 12.006001, 12.00060001  
 (a) 12 (b) 12

2. (a)



- (b) 7.2, 7.1, 7.01, 7.001, 7.0001  
 (c) 7 (d) 7

### Exercise 20.1

1. (a)  $5x^4$  (b)  $18x^2$   
 (c)  $-28x^3$  (d)  $8x^5$   
 (e)  $2x - 4$  (f)  $72x^8 - 30x$   
 (g)  $19 + 22x$  (h)  $6x + 5 - 21x^2$
2. (a)  $1 - 7x^6$   
 (b)  $40x - 9x^8$   
 (c)  $33x^2 - 18x - 7$   
 (d)  $30x^4 - 3x^2 - 13$   
 (e)  $10 - 18x - 4x^3$   
 (f)  $32x^3 - 15x^2 + 4x + 7$   
 (g)  $x + 3x^2 - \frac{10}{3}x^4$   
 (h)  $0.9x^2 + 0.24x - 1$

### Exercise 20.2

1. (a)  $-\frac{5}{7x^2}$   
 (b)  $10x^4 + 2x^{-3}$   
 (c)  $3x^2 + \frac{9}{4x^3}$   
 (d)  $1 + \frac{3}{2x^3}$   
 (e)  $-15x^{-6} - 22x$
2. (a)  $8x^7 - 5x^{-6}$   
 (b)  $32x^3 - 6x^2 - 4x^{-5} + 13$   
 (c)  $-\frac{3}{x^4} - 10$   
 (d)  $-\frac{35}{x^8} + 4$   
 (e)  $9 + \frac{3}{5x^2}$
3. (a)  $1 - 9x^2 + 25x^4 - 49x^6$   
 (b)  $8x^{-3}$   
 (c)  $10 - 27x^{-4} - 2x^{-2}$   
 (d)  $\frac{6}{7}x - \frac{10}{x^3}$   
 (e)  $-\frac{3}{x^4} + \frac{20}{x^6}$
4. (a)  $6x^2 - 14x - 4$   
 (b) (i)  $-12$   
 (ii) 48  
 (iii)  $-4$   
 (c) 16; gradient of the  $f(x)$  curve at the point where  $x = -1$
5. (a)  $8 - 6x^2$   
 (b) (i) 8  
 (ii)  $-16$   
 (iii)  $-46$

### Exercise 20.3

1. (a)  $10 \text{ m s}^{-1}$   
 (b)  $25 \text{ m s}^{-1}$   
 (c)  $13 \text{ m s}^{-1}$ ; velocity of the particle at  $t = 3$
2. (a)  $14 - 20t$   
 (b) (i) 4 (ii) 0
3. (a)  $6\pi = 18.8$   
 (b)  $10\pi = 31.4$



### Exercise 20.4

- (a) 1500 (b) -36 (c) -13  
(d) 18 (e) -4 (f) -42  
(g) 10 (h) -0.5 (i) -32  
(j) -10 (k) 3.20 (l) -1
- (a) (i)  $16q$  (ii) 160  
(b) (i)  $10q$  (ii) 1200  
(c) (i)  $5 + 6q$  (ii) 485  
(d) (i)  $6q - 10$  (ii) 74  
(e) (i)  $6q^2 - 18q + 45$  (ii) 236445
- (a)  $2x - 4$  (b) 2 (c) 6
- (a)  $3x^2 - 8x$   
(b) 28  
(c) (0.667, 6.52) and (2, 0)
- (a)  $2 - \frac{1}{x^2}$   
(b) 1  
(c) (-0.333, -3.67) and (0.333, 3.67)
- (a) \$115 (b) 45
- (a)  $28 - 20t$  (b) -20

### Exercise 20.5

- (a)  $y = 24x - 58$  (b)  $y = 6x + 13$   
(c)  $y = 4x + 12$  (d)  $y = 0.25x + 0.25$   
(e)  $y = 17x - 7$  (f)  $y = -0.25x + 1$   
(g)  $y = 0.75x + 5.75$  (h)  $y = -3.56x + 6.96$
- (a)  $y = -7x - 21$  (b)  $y = 13x - 56$   
(c)  $y = 42.5x + 0.25$  (d)  $y = -12x + 29$   
(e)  $y = -49x + 36$  (f)  $y = 5$
- (a)  $6x^2 - 2x + 4$  (b) 24  
(c) 21 (d)  $y = 24x - 27$
- (a)  $-\frac{x}{8}$  (b)  $\frac{1}{8} = 0.125$   
(c)  $\frac{143}{16} = 8.94$  (d)  $2x - 16y + 145 = 0$

### Exercise 20.6

- (a)  $y = -0.0417x + 38.2$   
(b)  $y = -0.167x - 5.5$   
(c)  $y = -0.25x + 7.75$   
(d)  $y = -4x + 2.375$   
(e)  $y = -0.0588x + 10.1$   
(f)  $y = 4x + 9.5$   
(g)  $y = -1.33x + 9.92$   
(h)  $y = 0.281x + 1.21$

- (a)  $y = 0.143x + 0.429$   
(b)  $y = -0.0769x - 29.8$   
(c)  $y = -0.0235x - 21.0$   
(d)  $y = 0.0833x - 7.25$   
(e)  $y = 0.0204x + 11.5$   
(f)  $x = -1$
- $y = -\frac{1}{3}x - \frac{4}{3}$  or  $y = -0.333x - 1.33$
- $y = -0.125x + 13.5$

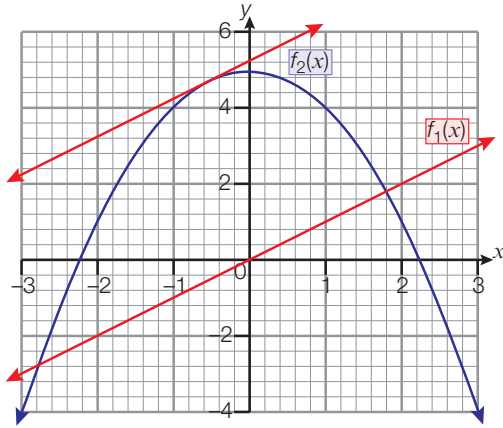
### Mixed examination practice 20

#### Exam-style questions 20

- $4x^3 - 21x^2 - 9$
- (a)  $2x - 8$  (b) -2  
(c) 7
- (a)  $\frac{dC_T}{dq} = 120 - 2q - 0.015q^2$   
(b) 16
- (a)  $-2 \text{ m s}^{-1}$  (b)  $-5 \text{ m s}^{-1}$   
(c)  $-6 \text{ m s}^{-1}$ ; velocity of the particle at  $t = 3.5$
- (a) (i)  $6t^2 - 8t + 4$   
(ii)  $12t - 8$   
(b) (i)  $12 \text{ m s}^{-1}$   
(ii)  $68 \text{ m s}^{-1}$   
(c) (i)  $4 \text{ m s}^{-2}$   
(ii)  $40 \text{ m s}^{-2}$   
(d)  $\frac{2}{3} \text{ s}$
- (a) -2; the  $y$ -coordinate of the point on the curve where  $x = 1$   
(b) 9; the gradient of the curve at the point where  $x = 1$   
(c)  $y = 9x - 11$
- (a) -6 (b)  $\frac{1}{6}$   
(c) 1 (d)  $y = -6x + 7$   
(e)  $y = \frac{1}{6}x + \frac{5}{6}$
- (a)  $3x^3 - 5x^2$  (b)  $9x^2 - 10x$   
(c) (i) 56 (ii)  $-\frac{1}{56}$   
(d) (i)  $y = 56x + 68$   
(ii)  $y = -\frac{1}{56}x - \frac{1233}{28} = -0.0179x - 44.0$

### Past paper questions 20

1. (a) (i)  $f_1'(x) = 1$   
 (ii)  $f_2'(x) = -2x$   
 (b)  $x = -\frac{1}{2}$   
 (c)



2. (a)  $6x^2 - 10x + 3$   
 (b) 7  
 (c)  $y = 7x - 11$
3. (a) 2  
 (b)  $f'(x) = 2x - 3$   
 (c) (2.5, -5.25)  
 (d) (1.25, -6.1875)  
 (e) (i) 1      (ii)  $y = x - 8$   
 (f)  $x = 1.5$   
 (g) (1.5, -6.25); gradient 0

## Chapter 21

### Exercise 21.1

1. Increasing for  $0 < x < 2$ ;  
 decreasing for  $x < 0$  and  $x > 2$
2. (a)  $x < -1.94$  and  $x > 0.943$   
 (b)  $-1.94 < x < 0.943$
3. (a) (i)  $x > 4$   
 (ii)  $x < 4$   
 (b) (i)  $x > -2.25$   
 (ii)  $x < -2.25$   
 (c) (i)  $x > 0.500$   
 (ii)  $x < 0.500$   
 (d) (i)  $x \in \mathbb{R}$   
 (ii) None

- (e) (i)  $x < -2$  and  $x > 2$   
 (ii)  $-2 < x < 2$   
 (f) (i)  $x < -1$  and  $x > 2$   
 (ii)  $-1 < x < 2$   
 (g) (i)  $x < 0$  and  $x > 6$   
 (ii)  $0 < x < 6$   
 (h) (i)  $x < -2.56$  and  $x > 1.56$   
 (ii)  $-2.56 < x < 1.56$

4.  $x < 0.268$  and  $x > 3.73$

5. (a)  $9 + 6x - 3x^2$   
 (b) -15  
 (c) Decreasing
6. (a)  $x^2 - 2x - 3$   
 (b) (i) 21  
 (ii) -4  
 (c) (i) Increasing  
 (ii) Decreasing

### Exercise 21.2

1. (a) Min (-2, -4)  
 (b) Max (4, 16)  
 (c) Min (3, -4)  
 (d) Max (1.5, 6.25)  
 (e) Max (-0.333, 0.185);  
 min (1, -1)  
 (f) Max (-1, 1); min (1, -3)  
 (g) Min (-1, 2); max (1, 6)  
 (h) Max (-0.5, 6); min (0.5, 4)  
 (i) Max (0.667, 4.19);  
 min (2, 3)  
 (j) Max (-1, 3); min (1, -5)
2. Stationary points and classification as in 1.  
 Gradient functions:  
 (a)  $2x + 4$       (b)  $8 - 2x$   
 (c)  $2x - 6$       (d)  $3 - 2x$   
 (e)  $3x^2 - 2x - 1$       (f)  $3x^2 - 3$   
 (g)  $3 - 3x^2$       (h)  $12x^2 - 3$   
 (i)  $3x^2 - 8x + 4$       (j)  $5x^4 - 5$
3. (a)  $\frac{dy}{dx} = 3x^2 - 6x - 8$   
 (b) P(-0.915, -6.96) and Q(2.92, -35.0)  
 (c) P - point ; Q - min point
4. (b) R: max; S: min
5. (a)  $3x^2 - 4$       (b)  $\pm 1.15$   
 (c), (d) Max (-1.15, 3.08);  
 min (1.15, -3.08)
6. Max (0, 0); min (4, -32)
7. Max (-1, 2); min (1, -2)

### Exercise 21.3

1. Length = width =

$$\frac{40}{3} = 13\frac{1}{3} = 13.3 \text{ cm;}$$

$$\text{height} = \frac{10}{3} = 3\frac{1}{3} = 3.33 \text{ cm}$$

2.  $6 \times 7 = 42$   
 3. 6 cm by 6 cm  
 4. 3 cm by 6 cm

### Exercise 21.4

1. (a)  $120 - 8q$  (b) 15  
 (c) £925,000
2. (a) 80  
 (b)  $\frac{dC}{dn}$  is negative to the left of  $n = 80$  and positive to the right of  $n = 80$ ; or  $\frac{d^2C}{dn^2} > 0$  at  $n = 80$ ; or the graph is a parabola that opens upward and so has a unique minimum point.
3. (b)  $12x^2 - 160x + 400$   
 (c)  $\frac{10}{3} = 3.33 \text{ cm}$   
 (d) Length = width =  $\frac{40}{3} = 13.3 \text{ cm;}$   
 height =  $\frac{10}{3} = 3.33 \text{ cm;}$   
 volume =  $593 \text{ cm}^3$
4. (c)  $2808 - \frac{27}{8}x^2$   
 (d) 28.84  
 (e) Length = 86.5 cm;  
 width = 28.8 cm;  
 height = 21.6 cm  
 (f)  $53997 \text{ cm}^3$
5. (a)  $320 - 8n$   
 (b) 40;  $\frac{dR}{dn}$  is positive to the left of  $n = 40$  and negative to the right of  $n = 40$ ; or  $\frac{d^2R}{dn^2} > 0$  at  $n = 40$ ; or the graph is a parabola that opens downward and so has a unique maximum point.  
 (c) \$6,400,000
6. (c)  $120\pi r - 6\pi^2r^2$   
 (d) 6.37 (e)  $2546 \text{ cm}^3$

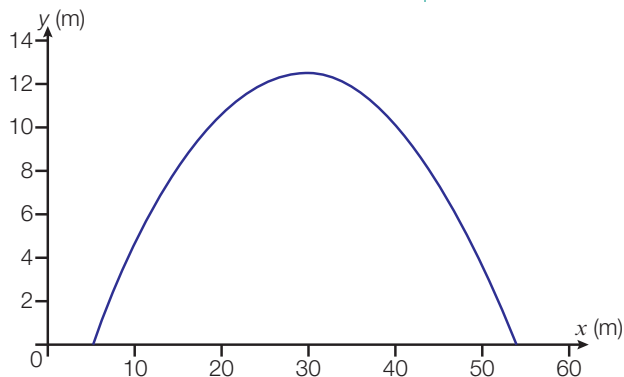
### Mixed examination practice 21

#### Exam-style questions 21

1. (a)  $x < 0.268, x > 3.73$   
 (b)  $0.268 < x < 3.73$
2. (a)  $21x^2 - 12$   
 (b) 324  
 (c) Increasing
3. (a)  $12x - 12x^2$   
 (c) Min (0, 3); max (1, 5)
4. (a)  $6x^2 - 18x - 24$   
 (b) -1 and 4  
 (c), (d) Max (-1, 16);  
 min (4, -109)
5. R(0, 5) and S(4, 37)
6.  $5000 \text{ m}^2$ ;  $50 \text{ m} \times 100 \text{ m}$
7.  $r = 3.74, h = 7.49$
8.  $200 \text{ m} \times 200 \text{ m} = 40\,000 \text{ m}^2$
9. Length = width = height = 20 cm; area =  $2400 \text{ cm}^2$
10. Length = width = 25.2 cm;  
 height = 12.6 cm;  
 area =  $1900 \text{ cm}^2$

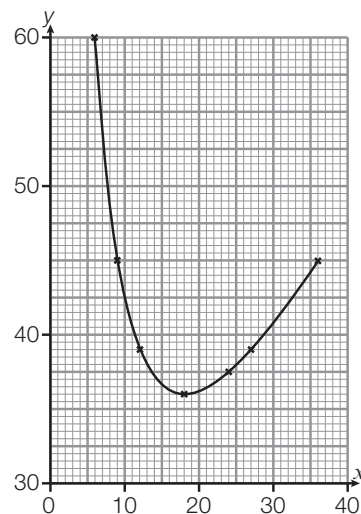
#### Past paper questions 21

1. (a)  $f'(x) = 2ax - 4x^{-2} = 2ax - \frac{4}{x^2}$   
 (b) -2
2. (a) 5.30  
 (b)  $-0.042x + 1.245$   
 (c) (i) 24.3 m (ii) 12.4 m  
 (d)



(e) 10.1 m

3. (b)  $1 - \frac{324}{x^2}$  (c) 18  
 (d) 36 (e)  $a = 36, b = 39$   
 (f)



- (g)  $x > 18$
4. (b)  $y = \frac{300 - 4x^2}{6x} = \frac{150 - 2x^2}{3x}$   
 (d)  $100 - 4x^2$   
 (e) (i)  $x = 5, y = \frac{20}{3}$   
 (ii)  $333\frac{1}{3} \text{ cm}^3$
5. (a)  $y = 2$   
 (b) Less than  
 (c) P is a local minimum

# Glossary

## A

**acceleration** the rate of change of velocity in relation to time

**algebra** a method of generalising problems in arithmetic

**algorithm** a systematic step-by-step process (a set of instructions) leading to a result

**angle of depression** angle between the horizontal and a lower line-of-sight line

**angle of elevation** angle between the horizontal and a higher line-of-sight line

**arithmetic progression/sequence** a list of numbers where the difference between consecutive numbers is constant

**arithmetic series** the sum of terms in an arithmetic sequence

**asymptote** a line approached by a curve but never reached

**axis (pl. axes)** a line which is used as a reference, e.g. on a graph

## B

**bar chart** diagram in which information is arranged into vertical or horizontal blocks

**bias** influence within a sample of data that favours a particular member(s) of a population

**biased sample** a sample which is limited to a particular, possibly unrepresentative group

**BIDMAS** the order of operations in working out an arithmetical expression (Brackets, Indices, Division, Multiplication, Addition, Subtraction)

**bivariate data** data showing the relationship between two variables

**boundary** used in a grouped frequency table, to describe the top or bottom values of one class

**box and whisker diagram** statistical diagram to display a five-figure summary

**break-even point** the point at which the cost of production and income are the same

## C

**chi-squared statistic ( $\chi^2$ )** used in a two-way table to test observed values against expected values

**chord** a straight line which joins two points on a curve

**coefficient** a number which is used to multiply a variable

**commission** amount (usually a percentage) charged by a financial institution for handling money

**common difference** the fixed difference between consecutive terms in an arithmetic sequence

**common ratio** the fixed multiplier from each term to the next in a geometric sequence

**complement of a set** element(s) which are not included in the set

**compound interest** a system in which interest is recalculated at regular intervals to include previous interest accumulated

**compound statement** a logical statement containing two or more propositions

**conditional probability** probability based on the assumption that an event has already occurred

**cone** a solid with one vertex and a circular base

**conjunction** a compound statement where two propositions are connected by 'and'

**connective** a symbol that links two propositions

**constant** a quantity with a fixed value

**continuous data** data that can be measured

**contradiction** a compound statement that is always logically false

**contrapositive** a new statement that combines the converse and inverse of an original logical statement

**converse** the reversal of two propositions in a logical statement

**coordinates** pairs of numbers which are used to uniquely locate a point on a graph

**correlation** the degree of association between two variables (can be positive or negative)

**cosine** a trigonometric ratio (usually abbreviated to 'cos');  $\frac{\text{adjacent}}{\text{hypotenuse}}$

**covariance** a measure of the connection between two variables

**cube** a solid in which the six faces are all squares

**cuboid** a solid in which the six faces are all rectangles

**cumulative frequency** the total frequency up to a certain data value

**cylinder** a solid prism with straight, parallel sides and a circular base

## D

- decimal places (d.p.)** the number of digits after the decimal point
- decreasing function** the section of the function where the gradient is always negative
- deflation** a progressive decrease in consumer prices
- degrees of freedom** in the chi-squared test, the number of pieces of independent data
- denominator** the bottom part of a fraction
- depreciation** a decrease in value due to age or other factors
- derived unit** a unit defined in terms of another unit
- difference** the result of subtracting a smaller number from a larger number
- differentiation** the process of finding the gradient function for any given function
- discrete data** data that can be counted
- disjunction** a compound statement where two propositions are connected by 'or'
- dispersion** the spread in a set of values
- displacement** the amount of movement of an object measured in a particular direction
- domain** the set of inputs into a function

## E

- empty (null) set** a set containing no elements
- equilateral triangle** a triangle in which all sides are the same length
- equivalence** a compound statement where two propositions are connected by 'if and only if'. Do not confuse this with **logical equivalence**
- estimate** to make a preliminary approximation
- event** in probability, an individual outcome or combination of outcomes being investigated, such as a particular result of rolling a die or choosing an object from a bag
- exchange rate** the relationship between the values of two different currencies
- exclusive disjunction** a compound statement where two propositions are connected by 'or ... , but not both'
- exponent** small number to the upper right of a number/letter (also called index or power)
- exponential function** a function of the form  $y = ka^n$
- exponential growth** growth in the form  $y = a^n$

## F

- factorise** to resolve an expression into a product of two or more factors, e.g.  $12 = 2 \times 6$ ;  $15x + 12y = 3(5x + 4y)$
- frequency table** a table that records the number of occurrences of an item or group of data
- function** the one-to-one, or many-to-one, relationship between two variables
- future value** a quantity used in financial calculations: the value of an investment after a certain number of years

## G

- general form** the most commonly written form of a formula
- geometric sequence** a sequence created by multiplying by the same value each time
- geometric series** the sum of terms in a geometric sequence
- gradient** the measure of the steepness of a slope
- grouped data** statistical data that has been put into groups, not listed individually

## H

- hemisphere** half of a sphere
- histogram** a diagram with the appearance of a bar chart in which the area of each bar (not the height) represents the frequency of the group of data
- hypotenuse** the side opposite the right angle in a right-angled triangle

## I

- implication** where two logical statements are connected by the words 'if ... then'
- improper fraction** a fraction in which the numerator is greater than the denominator
- included angle** angle between two given sides
- increasing function** the section of the function where the gradient is always positive
- index (indices)** small number to the upper right of a number/letter
- infinity** the concept of having no end
- inflation** a general increase in prices, and the corresponding decrease in purchasing power, over time
- integer** a whole number, which may be positive, negative, or zero

**intercept** point at which a line or curve crosses an axis; may be specifically referred to as  $x$ -intercept or  $y$ -intercept

**interest rate** percentage charged by a financial institution on a borrowed sum

**interquartile range** the value obtained by subtracting the lower quartile from the upper quartile

**intersection** (geometry) the point where two lines cross

**intersection** (set theory) the overlap of two or more sets

**interval** the space between two numbers or between two boundaries

**inverse** an opposite operation, e.g. adding and subtracting are inverse operations

**irrational number** a real number that cannot be written as a fraction, e.g.  $\pi$ ,  $\sqrt{2}$

**isosceles triangle** a triangle with two sides of equal length

## K

**kelvin** the SI unit of temperature;  $0^{\circ}\text{C} = 273.16\text{ K}$

## L

**limit** a value that is approached but not reached

**line of best fit** a line on a graph which shows a general trend

**line of symmetry** a line that cuts a figure into two parts that are mirror images of each other

**line segment** a line which has fixed end points

**linear equation** an equation with two variables which gives a straight line when plotted on a graph

**logical equivalence** is when two compound statements mean the same thing; the compound statements will have exactly the same final column in their truth tables. (Also known as 'logically equivalent'.) Do not confuse this with **equivalence**

**lump sum** in finance, money paid in a single payment, not in instalments

## M

**mapping** a relationship between two sets of numbers

**maximum (pl. maxima)** a stationary point where the gradient changes from positive to zero to negative

**mean** the sum of data values divided by the number of data values, usually denoted by the Greek letter  $\mu$  (population) or  $\bar{x}$  (sample)

**measure of central tendency** the mean, the median or the mode

**median** the middle number of a set of ordered data values

**midpoint** the halfway mark between two points

**minimum (pl. minima)** a stationary point where the gradient changes from negative to zero to positive

**mode (modal)** the most frequently occurring value (group) in a set of statistical data

**mutually exclusive** describes events that cannot happen at the same time

## N

**natural number** a number from the set of counting numbers: a whole number that is greater than or equal to zero

**negation** a statement of denial or contradiction, the assertion that a particular proposition is false

**normal** a line at right angles to the tangent of a curve

**null hypothesis** a statement asserting that there is no relationship between two variables

**number line** a line drawn to illustrate the order of real numbers

**numerator** the top part of a fraction

## O

**ogive** a distribution curve where the frequencies are cumulative

**optimisation** using calculus to find the best solution to a problem

**outcome** the result(s) of a probability experiment, such as obtaining a tail from one throw of a coin

**outlier** a value that lies a long way outside the general range of data

## P


**parabola** a  $\cup$  or  $\cap$ -shaped curve demonstrating a quadratic equation

**per annum** for each year, e.g. 5% interest per annum

**percentage error** the difference between an estimated value and the exact value, calculated as a percentage relative to the exact value

**perimeter** the length of the outline of a closed figure

**perpendicular** at right angles to a line or plane



**pie chart** diagram in which data is arranged as sectors of a circle, the angles of the sectors representing the frequency of the data

**polynomial** a sum of two or more terms in the form  $y = a + bx + cx^2 + \dots$

**population** the whole group that is being studied

**power** small number to the upper right of a number/letter (also called exponent or index)

**prime number** a number that has exactly two different factors, itself and 1

**prism** a solid whose cross-sections parallel to an end are all identical

**probability** the chance that an event will occur

**proportion** the relationship between two or more numbers, or between the parts of a whole

**proposition** a basic statement in logic; it can be true, false or indeterminate

**present value** a quantity used in financial calculations: the initial amount of an investment

**p-value** a measure of evidence against the null hypothesis

**pyramid** a solid with a polygonal base and an apex above the base. This is only correct for a right pyramid

**Pythagoras' theorem** the theorem for a right-angled triangle that links the length of the hypotenuse with the lengths of the other two sides

**Pythagorean triples** sets of three numbers that fit Pythagoras' theorem, e.g. 5, 12, 13

## Q

**quadratic equation** an equation in which the highest exponent of the variable is 2, i.e. it contains a square term:  $ax^2 + bx + c = 0$

**qualitative data** data that is neither counted nor measured

**quantitative data** data that is counted or measured

**quartile** the values that divide a set of data into four equal parts

**quotient** The 'whole-number' part of the result of dividing one number (or expression) by another

## R

**radius** the distance from the centre of a circle to the circumference

**random sample** a sample of subjects that is randomly selected from a group

**range (function)** the set of output values of a function

**range (statistics)** the difference between the highest and lowest values of a set of data

**ratio** the relationship between two different numbers or quantities, e.g. 5 : 6

**rational number** a number that may be expressed as a fraction, e.g.  $\frac{a}{b}$

**real number** any number that can be placed on the number line

**regression line** used to analyse information on a scatter diagram

**representative sample** a statistical sample that fairly represents all the data collected

**right cone/prism/pyramid** a figure where the apex is directly above the centre of its base

**right-angled triangle** a triangle with one right angle ( $90^\circ$ )

**root of an equation** the solution to a polynomial equation of any degree

**rounding** the approximation of a number to a given degree of accuracy

## S

**sample** a subset of a larger group

**sample space** the complete set of possible outcomes from an experiment

**sample space diagram** a diagram listing every result of a probability experiment

**scatter diagram** a graph which uses paired data to analyse the correlation between two variables

**scientific notation** writing very large or very small numbers in standard form

**sequence** an ordered list of numbers (that follow a 'rule')

**series** sum of a sequence

**set** a group of numbers or objects with a common characteristic

**SI unit** the international system of units; there are seven base units of measurement

**significant figures (s.f.)** the number of digits used to specify how precisely a value is expressed

**simple interest** a system where interest calculations are based only the original amount deposited or borrowed

**simultaneous equations** a set of equations for which a common solution is sought

**sine** a trigonometric ratio (usually abbreviated to 'sin');  $\frac{\text{opposite}}{\text{hypotenuse}}$

**skewed** slanted to one side

**slant height** distance from the apex of a cone or pyramid to a point on the perimeter of its base

**solid** three-dimensional figure

**solution** the answer to a problem; a value which, when substituted for the variable, makes the equation true

**sphere** solid on which all points on the surface are equidistant from the centre (a ball)

**standard deviation** the measure of the amount by which a set of values differs from the arithmetical mean

**standard form** a method of writing very large or very small numbers in a compact form, e.g. 25 000 000 in standard form is  $2.5 \times 10^7$  (also called scientific notation)

**statement** a proposition in logic

**stationary point** a point on a curve where the gradient of the curve is zero

**subset** a set contained within a larger set

## T

**tangent (graph)** a line which touches (but does not cross) a curve

**tangent (trigonometry)** a trigonometric ratio (usually abbreviated to 'tan');  $\frac{\text{opposite}}{\text{adjacent}}$

**tautology** a compound statement that is always true

**term** a number in a sequence, or an element of an algebraic expression that is separated from other elements by a + or – sign

**tree diagram** a branched diagram used to illustrate probabilities

**trend line** a line on a graph which shows a general trend

**trial and improvement** improving accuracy through repeated calculations

**trigonometric ratio** the ratio of two sides in any right-angled triangle

**trigonometry** In triangles, the study of angles and lines and their relationships

**truth table** a table for the study of logic that lists all possible combinations of True and False

**turning point** a point where a curve changes direction

## U

**union** contains all elements of two or more sets without repeats

**universal set** the set that includes all the elements that are under consideration

**unknown** a value represented by a letter

## V

**variable** a quantity that can change

**variance** the square of the standard deviation

**velocity** the rate of change of the displacement (distance) of an object as it moves in a particular direction

**Venn diagram** a diagram that uses circles to demonstrate the relationships between sets



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