Name: _____ Block: _____ Date: _____

Lesson 13.2 - The χ^2 Test for Independence

In Nariko's town, people say that it is better if you get Ms B as your examiner for the driving test; she is believed to pass more people than Miss C or Mr A.

Nariko decides to collect some actual results from the test centre, and analyse them to determine whether there is any truth to the common belief.

To check if pass/fail rates in the driving test are dependent on the examiner, Nariko uses a χ^2 test. First, she puts the data she collected in a two-way table:

Observed frequencies	Mr A	Ms B	Miss C	Total
Pass	28	38	35	101
Fail	20	10	18	48
Total	48	48	53	149

All χ^2 tests start from the assumption that the two factors being tested are independent. This independence is stated as the null hypothesis, usually denoted by H_0.

Step #1: State the null and alternative hypothesis.

Nariko's **null hypothesis** is:

H_0: The pass/fail rate is independent of the instructor.

In contrast, the **alternative hypothesis** is:

H_1: The pass/fail rate is dependent on the instructor.

Step #2: Calculate the Expected Frequencies.

To apply the χ^2 test, Nariko then needs to calculate the expected frequencies, which are the frequencies that should occur if the factors were truly independent.

Observed frequencies	Mr A	Ms B	Miss C	Total
Pass	28	38	35	101
Fail	20	10	18	48
Total	48	48	53	149

$$\text{expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{total}}$$

Expected Frequencies	Mr. A	Ms. B	Miss C
Pass	$\frac{101 \times 48}{149} = 32.5$	$\frac{101(48)}{149} = 32.5$	$\frac{101(53)}{149} = 35.9$
Fail	$\frac{48(48)}{149} = 15.5$	$\frac{48(48)}{149} = 15.5$	$\frac{48(53)}{149} = 17.1$

Step #3: Calculate the χ^2 Statistic using a table.

f_O	f_E	$f_O - f_E$	$(f_O - f_E)^2$	$\frac{(f_O - f_E)^2}{f_E}$
28	32.5	−4.5	20.25	.623
20	15.5	4.5	20.25	1.31
38	32.5	5.5	30.25	.931
10	15.5	−5.5	30.25	1.95
35	35.9	−.9	.81	.0226
18	17.1	.9	.81	.0474
			Total:	4.88

χ^2_{STAT}

Step #4: State your conclusion based off of a 5% significance level.

$df = (R-1)(C-1) = 1(2) = 2$

$\chi^2_{crit} = 5.991$

Because $\chi^2_{stat} < \chi^2_{crit}$, we can say that the pass/fail rate & instructor are independent of each other. Therefore we accept H_0.