

Lesson 13.2

The sum of an arithmetic sequence.

$$S_n = \frac{n}{2}(u_1 + u_n)$$

EXAMPLE #1

Find the sum if $u_1 = 35$ & $u_{32} = 376$.

$$S_n = \frac{32}{2}(35 + 376) = 16(411) = \boxed{6576}$$

The following formula can be used if you DO NOT know the last term.

$$S_n = \frac{n}{2} [2u_1 + (n-1)d]$$

EXAMPLE #2

Find the sum if $u_1 = 33$, $d = 16$ & $n = 18$.

$$S_n = \frac{18}{2} [2(33) + (18-1)(16)] = 9(66 + 272)$$
$$= \boxed{3042}$$

EXAMPLE #3

Find the sum: $145 + 95 + 45 + \dots$; 28 terms

$$u_1 = 145, \quad d = -50, \quad n = 28$$

$$S_n = \frac{n}{2} [2u_1 + (n-1)d]$$

$$= 14(290 - 1350)$$

$$= \boxed{-14840}$$

EXAMPLE #4

Consider the sequence consisting of all the odd numbers from 1 to 99:

1, 3, 5, 7, 9, ..., 97, 99

It is an arithmetic sequence with $u_1 = 1$ and $d = 2$.

a) How many terms are there?

b) What is the total if you add all the terms up?

a) $u_n = u_1 + (n-1)d$

$$99 = 1 + (n-1)(2)$$

$$99 = 1 + 2n - 2$$

$$99 = 2n - 1$$

$$100 = 2n$$

$$\boxed{50 = n}$$

b) Method #1

$$\left\{ \begin{aligned} S_n &= \frac{n}{2}(u_1 + u_n) \\ &= \frac{50}{2}(1 + 99) \\ &= 25(100) \\ &= \boxed{2500} \end{aligned} \right.$$

Method #2

$$\left\{ \begin{aligned} S_n &= \frac{n}{2}[2u_1 + (n-1)d] \\ &= \frac{50}{2}[2(1) + (50-1)(2)] \\ &= 25(2 + 98) \\ &= \boxed{2500} \end{aligned} \right.$$